

T-79.4501 Cryptography and Data Security (5 cr)

T-110.5210 Cryptosystems (5 cr)

Students of the course T-110.5210 Cryptosystems (4 cr): Give answers to at most four (4) problems. Please, mark clearly that your exam is for 4 credits only!

EXAM

Thursday, February 20, 2014

1. (6 pts) Consider an *Autokey cipher* in ring \mathcal{R} with key b of length $m = 1$. Then given plaintext $x = (x_1, \dots, x_n)$ the ciphertext $y = (y_1, \dots, y_n)$ is computed as

$$\begin{aligned}y_1 &= x_1 + b \\ y_i &= x_i + x_{i-1}, \text{ for } i = 2, \dots, n.\end{aligned}$$

The attacker sees the ciphertext, and transforms it to sequence $z = (z_1, \dots, z_n)$ as follows:

$$\begin{aligned}z_1 &= y_1 \\ z_i &= y_i - z_{i-1}, \text{ for } i = 2, \dots, n.\end{aligned}$$

Show that then z is equal to the ciphertext obtained by encrypting x using the 2-dimensional Vigenère cipher with keyword $(b, -b)$.

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (a) (2 pts) Show that A represents multiplication by an element α in the field $\mathbb{F} = \mathbb{F}_2[x]/\langle x^3 + x + 1 \rangle$, where $\alpha(x) = x + 1$
- (b) (2 pts) Calculate α^{-1} in \mathbb{F} using the Extended Euclidean Algorithm.
- (c) (2 pts) Give A^{-1} . (Hint: A^{-1} represents multiplication by α^{-1} in \mathbb{F} .)
3. (a) (3 pts) Describe the operation of the Counter (CTR) mode of a block cipher.
- (b) (3 pts) Block cipher in CTR mode can be regarded as a stream cipher. What is the length of its period in bits?
4. Consider the RSA cryptosystem with modulus $n = 31 \cdot 43 = 1333$.
- (a) (3 pts) The random number generator returns two numbers 245 and 143. Which of them is suitable to be used as a private decryption exponent d ?
- (b) (3 pts) Decrypt the ciphertext $c = 903$ with the help of the Chinese Remainder Theorem. That is, compute

$$\begin{aligned}m_1 &= c^d \bmod 31 \\ m_2 &= c^d \bmod 43\end{aligned}$$

and then use the Chinese Remainder Theorem to compute m such that

$$\begin{aligned}m_1 &= m \bmod 31 \\ m_2 &= m \bmod 43.\end{aligned}$$

5. (6 pts) The DSA signature is a pair (r, s) , where

$$\begin{aligned}r &= (g^k \bmod p) \bmod q \\ s &= (h(m) + dr)k^{-1} \bmod q.\end{aligned}$$

Alice uses a toy version of the DSA signature scheme with a prime modulus $p = 43$ and generator $g = 21$ of order $q = 7$. By accident, Alice generates signatures for two different messages with the same per-message random number k . The hashes of the two signed messages are 2 and 3, and the signatures are $(2, 1)$ and $(2, 6)$, respectively. Compute Alice's private key without computing a discrete logarithm.

Exam Calculator Policy. It is allowed to use a function calculator, however no programmable calculator.