1:	2:	3:	Extra:	**1-3 Total:**	**/ 21**
4:	5:	6:		**4-6 Total:**	**/ 15**

Aalto ME-C3100 Computer Graphics, Fall 2013

Lehtinen / Hölttä, Peussa

Tentti/Exam, February 19 2014

Allowed: Two two-sided A4 sheets of notes, calculators (also symbolic). Return your notes with your answers. Write your answers in either Finnish or English.

The following applies to students who are taking the class in fall 2013. If you have not taken the midterm (välikoe) in October 2013, you must answer all questions. Questions 1-3 cover the material from the 2nd half of class (after October's midterm). Questions 4-6 cover the material from the midterm. Only answer 4-6 if 1) you didn't take the midterm, or 2) wish to raise your midterm score. You're guaranteed not to make yourself worse off if you try (we'll scale the points accordingly).

Name, student ID: ______________________________

1 Rendering Basics [/ 6]

1.1 Ray Casting vs. Rasterization [/ 4]

Give pseudocode for rendering an image using a ray caster and a rasterizer.

Ray Casting | Rasterization

1.2 Working Set [/ 2]

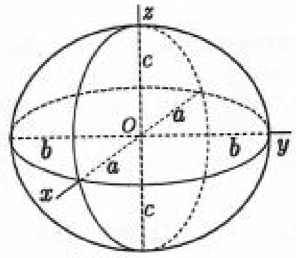
What are the main differences between the working sets in rasterization and ray casting? I.e., what needs to be kept in memory during execution in each case?

2 Ray Casting/Tracing [/ 9]

2.1 Ray Representation [/ 1]

What is the explicit representation of a ray in terms of its origin $\mathbf{o}$ and direction $\mathbf{d}$, in terms of a single scalar parameter t? Give a formula.

2.2 Ray-Ellipsoid Intersection [/ 6]



An axis-aligned, origin-centered ellipsoid is defined by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

where $a, b, c > 0$ are the length of the semi-axes.

a) What is special about the ellipsoid where $a, b, c > 0$ are all equal? [/ 1]

b) Derive the formula for intersecting a ray with the ellipsoid. The result is a quadratic equation in t with coefficients A, B, C. Like always, we will substitute the equation of the ray to the implicit equation of the ellipsoid.

b1) First, write the equation that defines the ellipsoid in the form $\mathbf{p}^{\mathrm{T}}\mathbf{M}\mathbf{p} = 1$, where $\mathbf{M}$ is a matrix that depends on a, b, c, and $\mathbf{p} = (x, y, z)^T$. Give the formula for the matrix $\mathbf{M}$. [/ 1].

b2) Then, using the above result, substitute $\mathbf{p}$ with the vector expression for the ray in terms of its origin, direction, and t. Expanding this out, find the coefficients A, B, C. *Keep your derivation in terms of matrices and vectors, this is much easier than writing out components! Also, you do not need to solve for the actual intersections given the coefficients, we assume the quadratic solution formula is known.* [/ 4]

2.3 Barycentric triangle representation [/ 2]

Give the barycentric representation of a triangle with vertices $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ in terms of $\{\alpha, \beta, \gamma\}$, including possible equality and inequality constraints.

3 Rasterization, Shading, Sampling [/ 9]

3.1 Edge Functions

Edge functions $e_i(x, y) = ax + by + c$ are 2D line equations that are computed from the three edges ($i = 1, 2, 3$) of a projected triangle. What is the mathematical condition that holds when a pixel/sample at (x, y) is inside the triangle? [/ 1]

3.2 Rasterization Using Edge Functions [/ 4]

Give pseudocode for rasterizing a triangle using edge functions. Start before projection. Your code should use screen bounding boxes for avoiding testing all pixels on the screen, and include z-buffering for visibility. You can assume the triangle has a constant color and that clipping has been performed already.

3.3 Irradiance [/ 2]

How does the irradiance incident on an *opaque (läpinäkymätön) surface* vary with the angle between the surface normal **n** and incident light direction **l**? *Account for both of the two cases: light above local horizon, and below it!*

3.4 The BRDF [/ 1]

The BRDF stands for "Bidirectional Reflectance Distribution Function". It is often denoted by $f_r(\mathbf{l}, \mathbf{v})$, where $\mathbf{l}$ is incident (light) direction and $\mathbf{v}$ is the outgoing (viewing) direction. What does the value $f_r(\mathbf{l}, \mathbf{v})$ tell you?

3.5 Shadow Rays [/ 1]

What is special about shadow rays, as opposed to rays cast from the camera, as far as intersection computations are concerned?

Midterm ends here!

4 Linear Algebra [/ 6]

4.1 Linearity [/ 2]

What properties characterize a linear function (operator) $L(x)$, with $x \in \mathbb{R}^n$? Write down one or two equations.

4.2 Affine Transforms [/ 2]

What happens to parallel lines under a linear transformation? How is the effect of an affine transform different as far as parallel lines are concerned, if at all?

4.3 Rotation Matrices [/ 2]

Rotation matrices are characterized by $\mathbf{R}^T\mathbf{R} = \mathbf{I}$ and $\det(\mathbf{R}) = 1$. Show that the matrix $\mathbf{RS}$ is a rotation matrix whenever $\mathbf{R}$ and $\mathbf{S}$ are rotation matrices. (Remember the elementary properties of determinants.)

5 Physically-Based Animation [/ 3]

5.1 Numerical Integration of ODEs [/ 3]

a) What do you hope to accomplish when you use stiff springs for modeling cloth? [/ **1**]

b) The explicit Euler method has problems with stiff springs. What is the typical manifestation of this? [/ **1**]

c) Why does it make sense to write higher-order ODEs as first-order systems? Give one reason.
[/ 1]

6 B-Splines [/ 6]

A cubic B-spline is a piecewise cubic curve defined by a "sliding window" of 4 control points. Each set of 4 contiguous control points defines one segment of the spline. Consider the 2-segment spline given by 5 control points $\{P_1, \ldots, P_5\}$. The ith segment ($i = 1, 2$) is given by $P_i(t) = \sum_{j=1}^{4} B_j(t)\, P_{i+j-1}$, where the basis functions are defined as $B_1(t) = \frac{1}{6}(1-t)^3$, $B_2(t) = \frac{1}{6}(3t^3 - 6t^2 + 4)$, $B_3(t) = \frac{1}{6}(-3t^3 + 3t^2 + 3t + 1)$, $B_4(t) = \frac{1}{6}t^3$. Show that cubic B-splines are C^2 continuous, i.e., that $P_1(1) = P_2(0)$, and that also the tangents and second derivatives of the segments P_1 and P_2 agree at the join.

A Extra Credit

No partial credit for extra credit questions. Extra credit is counted towards the score of questions 1-3 only.

A.1 Splitting Bézier Curves [/ 6]

Let's think about splitting a Bézier curve in two pieces at $t = 0.5$. Remembering the de Casteljau construction (see picture), it is clear that this can be accomplished by certain matrices. I.e., if the original curve has control points $\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3$ (column vectors), so that the geometry matrix is $\mathbf{G}_{\text{orig}} = (\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3)$, the new control points for the two resulting curves can be written as $\mathbf{G}_1 = \mathbf{G}_{\text{orig}}\mathbf{S}_1$ and $\mathbf{G}_2 = \mathbf{G}_{\text{orig}}\mathbf{S}_2$ with some (constant) matrices $\mathbf{S}_1$ and $\mathbf{S}_2$. Give the explicit expressions for $\mathbf{S}_1$ and $\mathbf{S}_2$.