T-61.5100 Digital Image Processing

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Answers can be given in English, Finnish or Swedish.
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1. Explain briefly, with 20–40 words or a mathematical definition, the following concepts or abbreviations: 6p.

(i) discrete two-dimension Fourier transform
(ii) unsharp masking
(iii) HSI color model
(iv) thresholding in image segmentation
(v) Wiener filter
(vi) duality of morphological dilation and erosion

2. Different types of distance measures will be studied in two-dimensional discrete space. (i) What conditions must a general function $D(\cdot,\cdot)$ fulfill in order to be a distance function or metric? (ii) Give the definitions of the Euclidan distance $D_e(p,q)$, the city-block distance $D_4(p,q)$, and the chessboard distance $D_8(p,q)$. (iii) Draw an image where you show all the points q of the two-dimensional discrete space for which either $D_e(o,q) = 5$, $D_4(o,q) = 5$ or $D_8(o,q) = 5$, where $o = (0,0)$ is the origin. Explain the result! (iv) Let three points in a two-dimensional discrete space be $a = (1,1)$, $b = (3,6)$ and $c = (6,2)$. Solve and illustrate with an image, which ones of the discrete points $p = (x,y) \in [0,6] \times [0,6]$ are such that p is closer to a than either b or c in the sense of the city-block distance:

$$D_4(p,a) < \min\left\{D_4(p,b), D_4(p,c)\right\} .$$

(v) Repeat the previous analysis using now the chessboard distance $D_8(p,q)$ instead of $D_4(p,q)$. (vi) Analyze the differences in the two results above. 6p.

Turn page!

3. We will study the use of image pyramids and wavelets in multiresolution processing of a 4×4 -sized image. Haar's scaling and wavelet functions are

$$\varphi(x) = \begin{cases} 1 & , \ 0 \leq x < 1 \\ 0 & , \ x < 0 \lor x \geq 1 \end{cases} \quad \text{and} \quad \psi(x) = \begin{cases} 1 & , \ 0 \leq x < 0.5 \\ -1 & , \ 0.5 \leq x < 1 \\ 0 & , \ x < 0 \lor x \geq 1 \end{cases}$$

(i) Construct a fully populated approximation pyramid for the image below by using averaging in 2×2 blocks. (ii) Construct a fully populated prediction residual pyramid corresponding to the approximation pyramid by using pixel replication. (iii) For the same image, calculate the two-dimensional discrete wavelet transform by using Haar's scaling and wavelet functions $\varphi_{0,0}(x)$, $\psi_{0,0}(x)$, $\psi_{1,0}(x)$ and $\psi_{1,1}(x)$. (iv) Repeat the previous calculation, now using Haar's scaling and wavelet functions $\varphi_{1,0}(x)$, $\varphi_{1,1}(x)$, $\psi_{1,0}(x)$ and $\psi_{1,1}(x)$. (v) Show and explain how the values of the approximation pyramid can be found also in the results of the wavelet transforms multiplied with constant values. (vi) Explain why the results of the wavelet transforms contain so many zeroes. Is that a good or bad phenomenon with respect to image compression? 6p.

$$f(x,y) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

4. Below there is one row of an image of 8 gray levels and width of 15 pixels. (i) Form a lossless run-length coding for the bit planes of the binary code. It is assumed that each row starts with a 0-valued run and that all run lengths are coded with four bits. (ii) Similarly, form lossless run-length coding based on the bit planes of Gray code. (iii) Calculate the average number of bits needed per pixel for both codings. Also calculate the compression ratios relative to the original representation. (iv) Calculate the relative redundancy of the original representation relative to the better run-length coding. (v) Evaluate the results. How could this coding be further enhanced? (vi) That type of redundancy is here being removed and what other types of redundancy do exist? 6p.

0 0 0 1 1 2 2 5 4 4 7 7 6 6 6