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T-79.4101 Discrete Models and Search (5 cr), Exam April 10, 2014

Write down on each answer sheet:
- Your name, degree program, and student number
- The text: "T-79.4101 Discrete Models and Search 10.04. 2014"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English. Calculators are allowed.

1. Which of the following claims are true and which are false?
   For each claim, justify your answer using 1-3 sentences.

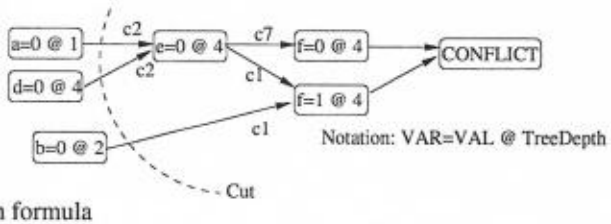
   (a) Complete search can be used for proving non-existence of solutions. *(2 points)*

   (b) Given a TSP instance of n cities $\{1,\ldots,n\}$ with a (nonnegative) distance matrix $D=(d_{ij})$, let $d(\pi)$ denote the cost of a complete tour π. The following inequality holds for any π: *(2 points)*

   $$d(\pi) \geq \frac{1}{2}\sum_{j} \min_{i,k:i\neq k} (d_{ij}+d_{jk}).$$

   (c) The following CSP is hyper-arc consistent: $\langle C_1(x,y,z), C_2(x,z); x\in\{1,2\}, y\in\{1,2\}, z\in\{2,3\}\rangle$, where $C_1=\{(1,1,2),(1,2,3),(2,1,3)\}$ and $C_2=\{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)\}$. *(3 points)*

   (d) The learned clause from the cut represented using the dotted line in the implication graph presented below is $a\vee d$. *(3 points)*



2. (a) Translate the Boolean formula *(5 points)*

   $$(\neg x_1 \rightarrow x_2 \vee x_3) \wedge (\neg x_2 \rightarrow \neg x_1 \vee \neg x_3) \wedge (x_1 \rightarrow \neg x_2 \vee x_3) \wedge (x_1 \wedge x_2 \rightarrow \neg x_3) \wedge (x_1 \wedge \neg x_2 \rightarrow x_3)$$

   into an equivalent set of clauses Σ, and trace an execution of the DPLL algorithm for Σ. In the execution, the order of variables considered for branching is x_1, x_2, x_3. The algorithm has to first consider a value *true* and then *false* during the splitting operation. Draw a search tree that shows the execution of the algorithm. Label the nodes in the tree with the literals obtained by unit propagation or by splitting rule.

   (b) Express the condition "if $x_1+2>0$ then $x_2-4\geq 0$" as a set of linear constraints, where x_1, x_2 are integers such that $-10\leq x_1, x_2 \leq 10$. *(5 points)*

3. Consider the following integer programming problem:

   $$\begin{aligned} &\max\ -2x_1+5x_2 \text{ s.t.}\\ &x_1-3x_2\geq -3\\ &x_1+x_2\leq 6\\ &x_2\geq 0\\ &x_1, x_2 \text{ are integers}\end{aligned}$$

   Give the linear relaxation of the problem, transform the linear relaxation to the Simplex tableau form and give a basic feasible solution for the relaxation in the Simplex tableau form. Is the solution you gave optimal? Justify your answer using the Simplex tableau. *(10 points)*

4. Consider the following optimization version NP-complete DOMINATING SET problem: *(10 points)*

   INSTANCE: Undirected graph $G=(V,E)$.
   QUESTION: Find a minimum size subset of nodes (vertices) of G, $D\subseteq V$, such that each node in V either is a member of D or has at least one neighbor in D, i.e., if $u\in V\setminus D$, then there exists an edge $\{u,v\}\in E$ such that $v\in D$.

   Describe a tabu search method for DOMINATING SET. Describe particularly clearly: (a) what are the candidate solutions considered and how does one choose an initial solution, (b) what is the neighborhood relation and how does one choose the next solution for consideration from the neighborhood of a given candidate solution, (c) which information is recorded in the tabu list, what is an aspiration rule and how is it used in the algorithm.