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T-79.4101 Discrete Models and Search (5 cr)
Exam December 17, 2014

Write down on each answer sheet:
- Your name, degree program, and student number
- The text: "T-79.4101 Discrete Models and Search 17.12.2014"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English. Calculators are allowed.

1. Consider the following NP-complete NUMBER SET BIPARTITION problem:

   INSTANCE: A set of $2n$ natural numbers $A = \{a_1, \ldots, a_{2n}\} \subseteq \mathbb{N}$.
   QUESTION: Is there a subset $B = \{b_1, \ldots, b_n\} \subseteq A$ containing exactly half the numbers and such that

   $$\sum_{i=1}^{n} b_i = \frac{1}{2} \sum_{j=1}^{2n} a_j \quad ?$$

   Formulate this task as an optimization problem by designing an appropriate objective function, and present some local search heuristic to search for good bipartitions according to your criteria. Describe particularly clearly: (a) what are the candidate solutions considered by your method and what is their neighborhood relation, (b) how does one choose the next solution for consideration from the neighborhood of a given candidate solution, and (c) how does one choose the initial candidate solution for the computation.

2. (a) Translate the Boolean formula

   $$(x \leftrightarrow y \wedge z) \wedge (y \leftrightarrow v \vee \neg w) \wedge (z \leftrightarrow \neg v \vee w) \wedge x$$

   into an equivalent set of clauses Σ, and trace an execution of the DPLL algorithm for Σ. In the execution, the order of variables considered for branching is x, y, z, v, w. The algorithm has to first consider a value *true* and then *false* during the splitting operation. Draw a search tree that shows the execution of the algorithm. Label the nodes in the tree with the literals obtained by unit propagation or by splitting rule.

   (b) Express the condition "if $x_1 + 2 > 0$ then $x_2 - 4 \geq 0$" as a set of linear constraints, where x_1, x_2 are integers such that $-10 \leq x_1, x_2 \leq 10$.

3. Consider the following integer programming problem:

   $$\begin{aligned}
   &\max\ 2x_1 - x_2 \text{ s.t.}\\
   &-2x_1 + 6x_2 \geq -6\\
   &x_1 + 3x_2 \leq 6\\
   &x_1 \geq 0\\
   &x_1, x_2 \text{ are integers}
   \end{aligned}$$

   Give the linear relaxation of the problem, transform the linear relaxation to the Simplex tableau form and give a basic feasible solution for the relaxation in the Simplex tableau form. Is the solution you gave optimal? Justify your answer using the Simplex tableau.

4. Consider the following optimization version of the NP-complete LONGEST CYCLE problem:

   INSTANCE: A directed graph $G = (V, E)$ and for each edge $e \in E$ a length $l(e) \geq 0$.
   QUESTION: Find a cycle that visits all nodes exactly once and has maximum length?

   Present in pseudocode form a branch-and-bound algorithm for this task. Describe particularly clearly: (a) what are the partial solutions and how is the algorithm initialized (b) how does the branching operation work (how are partial solutions extended) (c) what is your bounding heuristic and how is it used to prune the search. Try to use a non-trivial bounding heuristic that promises effective pruning.

Grading: Each problem 10p, total 40p.