T-61.3025 Hahmontunnistuksen perusteet

Exam 19. 5. 2014

1. Assume two classes in 2-dimensional feature space, where the density functions of the classes are uniform; the density of class 1 is uniform (constant) in the square whose corners are $(1,1),(1,-1),(-1,1),(-1,-1)$ and the density of class 2 is uniform in the rectangle whose corners are $(2,-1),(-1,2),(4,1),(1,4)$. The prior probabilities are the same. Plot a picture.

   a) What are the Bayes optimal decision regions for the 2 classes?

   b) What is the classsification error of the Bayes classifier?

2. Develop the Parzen estimate for a one-dimensional density function $p(x)$ using the following sample x_i:

   2.5, 2.8, 3.4, 4.2, 4.5, 4.7, 5.2, 5.6, 7.5.

   Choose the Parzen window function as a triangle whose two sides have equal lengths and the length of the base is chosen suitably. No mathematical expression is needed, it is enough to plot the solution as a picture.

3. Explain the support vector machine: what is it used for, and what is the principle of building it for a given training set?

4. Apply the c-means algorithm for the data set

   (0,0), (1,1), (2,2), (4,3), (4,4), (5,3), (5,4), (6,5).

   Choose the number of clusters as c=2. Also, plot a picture of how your algorithm is working.

5. Let us try to produce squares using a grammar whose terminal symbols are a line of unit length to the right o, a line of unit length downwards a, negation $\neg$ and concatenation $+$. For example a unit square is $o + a + \neg o + \neg a$. Non-terminals are

$$Square, Side1, Side2, Side3, Side4$$

   and the production rules are

$$\begin{aligned} Square &\rightarrow Side1 + Side2 + Side3 + Side4 \\ Side1 &\rightarrow o \mid Side1 + o \\ Side2 &\rightarrow a \mid Side2 + a \\ Side3 &\rightarrow \neg o \mid Side3 + \neg o \\ Side4 &\rightarrow \neg a \mid Side4 + \neg a \end{aligned}$$

   a) Is the grammar able to produce squares?

   b) Show that the grammar produces also something else than just squares.

   c) How should the grammar be changed in order to make it produce only squares?