CS-E5750 Nonlinear Dynamics and Chaos
Exam 6.4.2017.

Calculator is allowed, no other material.

Problem 1. Analyse the following system. Sketch the vector fields as r is varied. Determine the critical value where the bifurcation occurs. Sketch the bifurcation diagram and determine which bifurcation is in question.

$$\dot{x} = rx - 4x^3$$

Problem 2. A simple model of a fishery is provided by the equation

$$\dot{N} = rN\left(1 - \frac{N}{K}\right) - H.$$

N is the size of the fish population (the number of fish, $N \geq 0$), r the growth rate, and K the carrying capacity. The effects of fishing are modeled by the term $-H$, where H is a constant.
(a) By defining $x = \frac{N}{K}$, $\tau = rt$, and $h = \frac{H}{rK}$, show (in sufficient detail) that the system can be written in dimensionless form as

$$\frac{dx}{d\tau} = x(1-x) - h.$$

(b) Plot the vector field for different values of h.
(c) Show that a bifurcation occurs at a certain value $h = h_c$ and classify the bifurcation.
(d) Describe (interpret) the long-term behaviour of the fish population for $h < h_c$ and $h > h_c$.

Problem 3. Show that a standard analysis of the system

$$\begin{aligned} \dot{x} &= -y - x^3 \\ \dot{y} &= x \end{aligned}$$

predicts a linear center at the origin. Then refine your analysis by transforming the dynamic equations to polar coordinates in order to take into account the effect of nonlinear terms. Any change?

Problem 4. Analyse the map

$$x_{n+1} = \frac{2x_n}{1+x_n}.$$