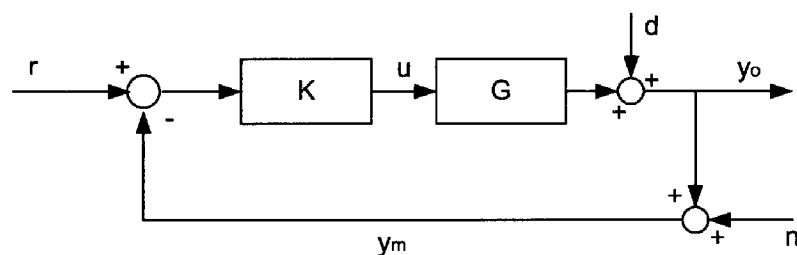


## ELEC-E8116 Model-based control systems

Full exam 15. 5. 2017

- Write the name of the course, your name, your study program, and student number to each answer sheet. *five*
- There are ~~three~~ (5) problems and each one must be answered.
- No literature is allowed. A function calculator can be used.
- Your solutions must contain enough information to show how you have solved the problems.

1. Consider the multivariable closed-loop control configuration



Write the equations describing the system and identify

- closed-loop transfer function
- sensitivity function
- complementary sensitivity function

Write expressions for the output variable  $y_0$ , control variable  $u$  and error variable  $e = r - y_0$ . What conditions to the above functions (a-c) should be set, in order the system to operate "well"?

2. For the system

$$\dot{x}(t) = x(t) + u(t), \quad x(0) = x_0$$

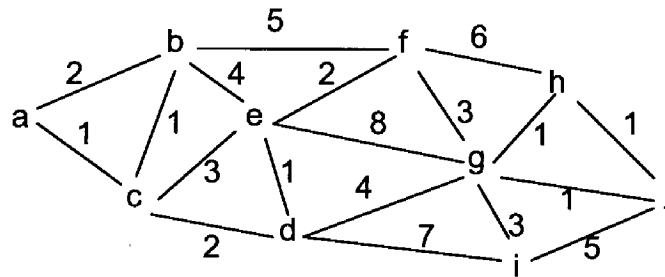
calculate the control law and optimal cost, when the criterion to be minimized is

$$J = \frac{1}{2} x(1)^2 + \int_0^1 u^2(\tau) d\tau$$

3. Consider the matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ .

- Calculate the *singular values* of the matrix.
- Explain, what *singular value decomposition* means. How can it be used to analyse multivariable systems?

4. The numbers in the below figure denote costs when moving from one node to another. Movement is possible only from the left to the right.



- Determine the minimum cost path for movement from node  $a$  to node  $j$ . Use dynamic programming.
- Explain how the Principle of Optimality is linked to dynamic programming.

5. Consider a multivariable system with the transfer function matrix

$$G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{3}{s+2} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

Calculate the poles and zeros of the system. Do the results imply any fundamental restrictions to control performance? If the answer is yes, explain what kinds of restrictions are there.

**Some formulas that might be useful:**

$$\dot{x} = Ax + Bu, \quad t \geq t_0$$

$$J(t_0) = \frac{1}{2} x^T(t_f) S(t_f) x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} (x^T Q x + u^T R u) dt$$

$$S(t_f) \geq 0, \quad Q \geq 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q, \quad t \leq T, \quad \text{boundary condition } S(t_f)$$

$$K = R^{-1}B^T S, \quad u = -Kx, \quad J^*(t_0) = \frac{1}{2} x^T(t_0) S(t_0) x(t_0)$$

$$\int_0^\infty \log |S(i\omega)| d\omega = \pi \sum_{i=1}^M \text{Re}(p_i)$$

$$|W_r(p_i)| \leq 1 \Rightarrow \omega_0 \geq \frac{p_i}{1 - 1/T_0}$$

$$|W_s(z)| \leq 1 \Rightarrow \omega_0 \leq (1 - 1/S_0)z$$