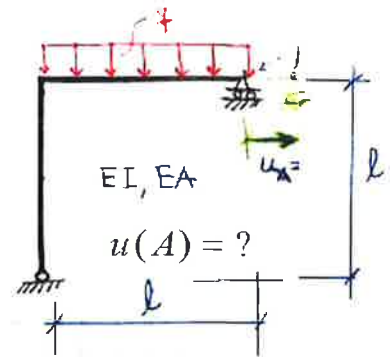


CIV-E1020 - Mechanics of Beam and Frame Structures

duration: 3h (given formulary on the verso of this paper)

Examination: 25.10.2016 (1st period: 12.09.2016-21.10.2016)

1. **Virtual force principle:** a) write mathematically the *dummy unit-load principle* for determining the horizontal displacement at roller support A of the plane frame accounting for both bending and stretching [1p].  
b) Assuming linear-elastic material and using the answer of question 1) determine this horizontal displacement and show explicitly the contributions from bending and stretching [4 p].



**1X: [Extra, 1p] - Strain energy of a frame:** write mathematically the expression of strain energy of a linear elastic frame accounting for bending, stretching and shearing a) in terms of internal forces [0.5 p] and b) in terms of corresponding generalized deformations [0.5 p]. (No need to account for 2<sup>nd</sup> order effects). /extra means that you can freely choose not to answer without losing points. However, right answer will count/

**2X: [Extra, 1p] - Principle of virtual work:** write mathematically the *virtual displacement principle* (the theorem) accounting only for bending of a simply supported beam (vapaasti tuettun kaksitukisen palkin taivutsu) with transversally distributed load. Only a complete right answer (the full proposition) will lead to the full extra-point.

2. **Structural analysis using the General Force Method**

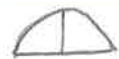
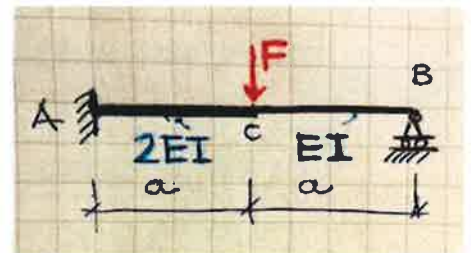
What is the degree of statical indeterminacy? (Redundancy degree).

Solve the reaction at the roller support B? [3p]

Determine the bending moment M and draw accurately the diagram (the graph). [2p]

Assume a linear-elastic material. Account only for bending when computing the flexibility coefficients (the Mohr's integrals).

Extra: determine the vertical deflection at C. [1p extra]



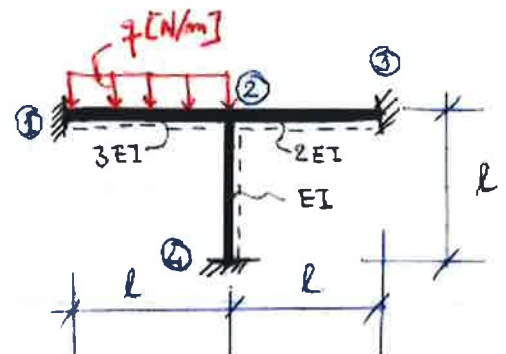
3. **Structural analysis using the Slope-Deflection Method** (displacement method)

What is the degree of kinematical indeterminacy? (Number of independent degrees of freedom).

Determine the rotation at node 2 [4p]

Solve the bending moment at support 3 [1p]

Assume a linear-elastic material and neglect the stretching effects on displacements and rotations.



4. **Structural analysis - Plastic limit load**

Use the kinematic method and

a) determine the plastic limit load and the failure mechanism. [4p]

b) check that the bending moment corresponding to the failure mechanism fulfils the yield condition (draw the bending moment at failure) [1p].

The material is elastic-perfectly plastic and the full plastic bending moment capacity is as shown in the figure and, notice that it changes in the two parts 1-2 and 2-3.

