Aalto University
School of Science

PHYS-E0413 Theoretical Mechanics
Midterm exam Friday 27.10.2017

1. Explain following concepts biefly (max 1p each)
   (a) Holonomic constraint
   (b) Action
   (c) Hamilton's principle
   (d) Lagrange equation
   (e) Generalized momentum
   (f) What does it mean if we say lagrangian has a symmetry?

2. Pendulum of length l and mass m (string/rod of the pendulum is massless) hangs from the roof of a moving car. Car is accelerated (on a horizontal track) with acceleration a from the initial velocity v_0. (Gravity acts in vertical direction with acceleration g.) Use the pendulum angle θ as the generalized coordinate.
   a) Find the Lagrangian for the pendulum $L(\theta, \dot{\theta}, t)$. (2p)
   b) Derive the equation of motion. (2p)
   c) What is the equilibrium angle of the pendulum? Discuss to what extent you can monitor the velocity and acceleration of the car by observing the behavior of the pendulum? (2p)

3. Dynamical system (q_1 and q_2 are coordinates, k_1 is a constant and $f(q_1)$ is a function of q_1) has a Lagrangian
$$L = \dot{q_1}^2 + f(q_1)\dot{q_2}^2 + k_1 q_1^2. \tag{1}$$
   a) Determine the Hamiltonian. (3p.)
   b) Find the Hamilton's equations of motion for the system. (2p.)
   c) Identify possible cyclic coordinates and associated conserved quantities. (1p.)

4. For notational simplicity assume we have only one generalized coordinate q and generalized momentum p. Poisson bracket of two functions of (q, p) is defined as
$$\{f_1, f_2\} \equiv \frac{\partial f_1}{\partial q}\frac{\partial f_2}{\partial p} - \frac{\partial f_1}{\partial p}\frac{\partial f_2}{\partial q}. \tag{2}$$
   a) Show that the time-evolution of function $f(q, p, t)$ is given by $df/dt = \{f, H\} + \partial f/\partial t$, where H is the Hamiltonian. (3p.)
   b) Transformation between coordinates and momenta $(q, p) \rightarrow (Q, P)$ (assume again only one pair for simplicity) is canonical if it preserves the Poisson bracket between coordinates and momenta. If $Q = q^n$ and $P = (p/2q)^m$, how could you choose n and m for the transformation to be canonical? (3p.)

Remember to answer in english unless you have a special permission to use some other language. Write your name, student number, study program, course code, and the date in all your papers. Use of calculators is forbidden.