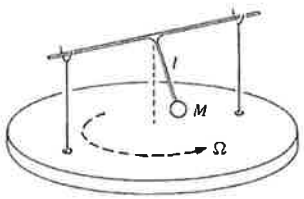
Aalto University
School of Science

PHYS-E0413 Theoretical Mechanics
Midterm exam Friday 28.10.2016

1. Explain following concepts biefly (max 1p each)
   (a) Holonomic constraint
   (b) Virtual displacement
   (c) Action
   (d) Cyclic coordinate
   (e) Hamilton's equations
   (f) Phase space

2. A pendulum of mass M and length l is attached to a horizontal rod that is in turn attached from both ends to a carousel rotating at an angular velocity
   (a) Write Langrangian for the pendulum (3p).
   (b) By assuming a small oscillation angle derive the oscillation period of the pendulum. At what angular velocity will the motion cease to be harmonic (i.e oscillatory)? (2p)
   (c) What happens for faster rotations and what does it imply for your assumption about small oscillation angles? (1p)



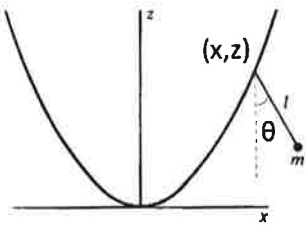
3. Show that constructing a new lagrangian L' from earlier one $L(q_1 \dots q_n, \dot{q}_1 \dots \dot{q}_n, t)$ by

$$L'(q_1 \dots q_n, \dot{q}_1 \dots \dot{q}_n, t) = L(q_1 \dots q_n, \dot{q}_1 \dots \dot{q}_n, t) + \frac{dF(q_1 \dots q_n, t)}{dt}, \qquad (1)$$

   gives rise to same equations of motion.

4. Pendulum is suspended from a parabola $z = ax^2$ so that it can move on it. (System experiences gravity downwards.) By using the point of suspension and pendulum angle as generalized coordinates

   a) Find the generalized/canonical momenta in terms of velocities (2p)
   b) Find the Hamiltonian (4p)
   Tip: You might want to define some functions to simplify your notation early on. Otherwise the expressions become long.



Remember to answer in english unless you have a special permission to use some other language. Write your name, student number, study program, course code, and the date in all your papers. Use of calculators is forbidden.