MS-E2140 Linear Programming
Final Exam **26.10.2017**

The exam is three hours long and consists of 4 exercises. The exam is graded on a scale of 0-25 points, and the points assigned to each question are indicated in parenthesis.

Problem 1

Solve the following LP problem by using the Two-Phase Simplex method (6pt).

$$
\begin{aligned}
\text{(P)} \quad \text{Minimize} \quad & 2x_1 \\
\text{s.t.} \quad & x_1 - x_3 = 3 \\
& x_1 - x_2 - 2x_4 = 1 \\
& 2x_1 + x_4 \le 7 \\
& x_1, x_2, x_3, x_4 \ge 0 \ .
\end{aligned}
$$

Problem 2

Are the following statements true (T) or false (F). Each correct answer gives +0.5 points and each incorrect answer gives −0.5 points. You don't need to justify your answers. The total number of points will be in the range $[0, 7]$ (7pt).

Consider an LP problem in standard form with m constraints and n variables.

(a) A basic feasible solution can have more than m positive components.

(b) A basic feasible solution can have fewer than m positive components.

(c) A basic solution can have negative components.

(d) A basic solution can have more than $n - m$ zero components.

(e) A basic solution can have fewer than $n - m$ zero components.

(f) Let $\mathbf{x}$ and $\mathbf{p}$ be solutions to the LP and its dual problem, determined by the same basis $\mathbf{B}$. Then $\mathbf{x}$ and $\mathbf{p}$ satisfy the complementary slackness conditions.

(g) If $\mathbf{x}$ and $\mathbf{p}$ satisfy the complementary slackness conditions, then they are optimal solutions.

(h) If the LP problem is unbounded, its dual problem must be infeasible.

(i) If the LP problem is infeasible, its dual problem must be unbounded.

(j) If the dual of the LP problem is infeasible, then the primal problem can be infeasible.

(k) If the LP problem has an optimal solution, its dual problem is feasible.

(l) The reduced cost of a basic variable can be zero.

(m) The reduced cost of a non-basic variable can be zero.

(n) If the primal basic solution is degenerate, then also the corresponding dual solution is degenerate.

Problem 3

Consider the following LP:

$$
\begin{array}{lll}
\text{(P)} & \text{Minimize} & -x_1 - 2x_2 + 4x_3 + 5x_5 \\
& \text{s.t.} & x_1 + x_3 - 2x_4 - x_5 + 2x_6 = 3 \\
& & x_1 + x_2 - x_4 - 3x_5 + 3x_6 + x_7 = 7 \\
& & -x_1 - 2x_2 - x_3 - x_4 + 5x_5 - 2x_6 \leq 4 \\
& & x_1, ..., x_7 \geq 0 \ .
\end{array}
$$

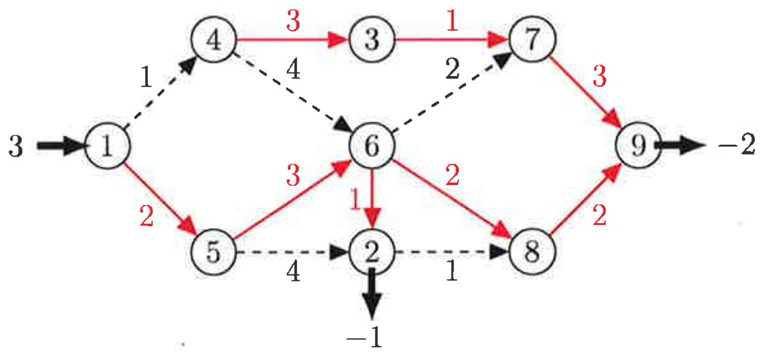
(a) Write the dual of P. (2pt)

(b) State the complementary slackness theorem. (2pt)

(c) Use the complementary slackness theorem to verify *if* the solution $x_1 = 3, x_2 = 4$, $x_j = 0$, $j = 3, ..., 7$ is optimal for P and justify your answer. (2pt)

Problem 4

Consider the uncapacitated network flow problem defined on the directed graph below. The label next to each arc (i, j) is its cost c_{ij}.



Consider the tree T defined by the solid red arcs in the figure and the associated tree solution. Apply the Network Simplex algorithm starting from the tree solution defined by T to find an optimal tree solution (6pt).

(*i*) Indicate the set T and report the arc flows at each iteration

(*ii*) Report the dual variables at each iteration

(*iii*) Explain how the leaving and entering variables are selected

(*iv*) Explain how the flows are updated