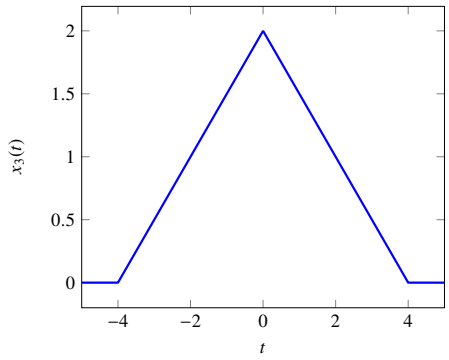
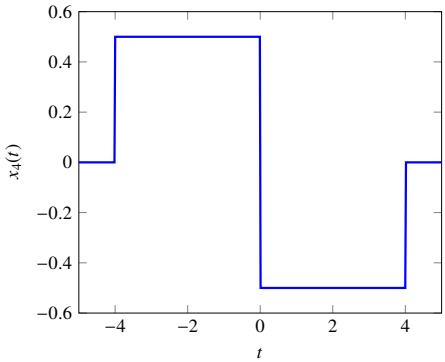
1. (a)

$$\langle x_1(t) - x_2(t), x_1(t)\rangle = \langle x_1(t), x_1(t)\rangle - \langle x_2(t), x_1(t)\rangle = |x_1(t)|^2 = 1$$
$$\langle x_1(t) - x_2(t), x_2(t)\rangle = \langle x_1(t), x_2(t)\rangle - \langle x_2(t), x_2(t)\rangle = -|x_2(t)|^2 = -1$$

(b)

$$x_4(t) = \frac{d}{dt} 2 \cdot \mathrm{tria}(t/4) = \frac{\mathrm{rect}\left(\frac{t+2}{4}\right) - \mathrm{rect}\left(\frac{t-2}{4}\right)}{2}$$

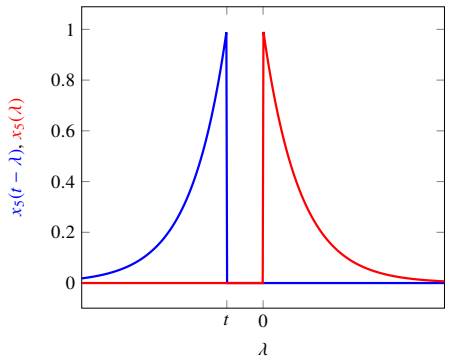


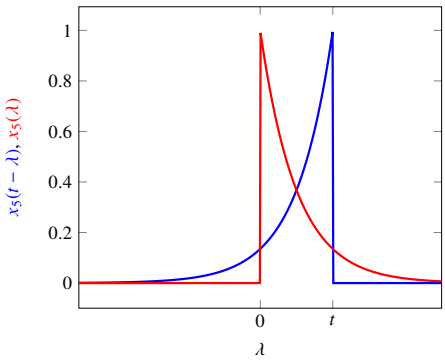


(c)

$$\int_{-\infty}^{\infty} x_3(t)\delta(t-2)dt \approx \int_{-\infty}^{\infty} x_3(2)\delta(t-2)dt = x_3(2)\int_{-\infty}^{\infty}\delta(t-2)dt = x_3(2) = 1$$

(d) Konvoluution laskeminen jakautuu kahteen alueeseen.

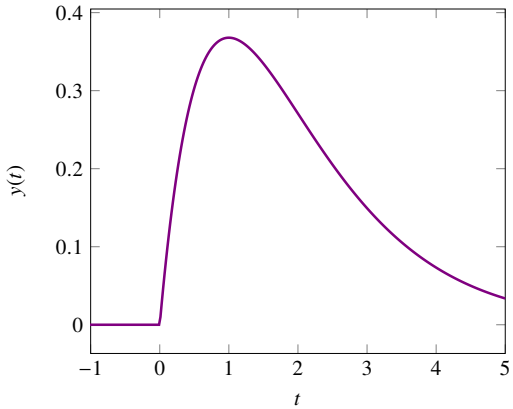




Kun $t \leq 0$ pulssit eivät ole limittäin, niiden tulo on nolla, ja konvoluutio $y(t \leq 0) = 0$.

Kun $t > 0$ pulssit ovat päällekkäin välillä $[0, t]$, eli niiden tulo poikkeaa nollasta vain tällä alueella.

$$y(t > 0) = \int_{-\infty}^{\infty} x_5(t-\lambda)x_5(\lambda)d\lambda = \int_0^t e^{-(t-\lambda)}e^{-(\lambda)}d\lambda = \int_0^t e^{-t}d\lambda$$
$$= e^{-t}\int_0^t \lambda = te^{-t}$$
$$y(t) = te^{-t} \cdot u_H(t)$$



2. (a)

$$2\pi f t = 20\pi t$$

$$f = 10\text{ Hz} \quad \text{ja} \quad T_0 = 1/f = 100\text{ ms}$$

(b)

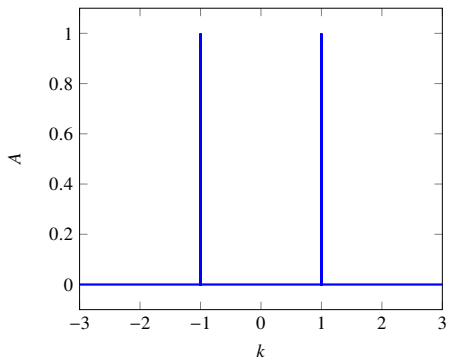
$$x_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-2\pi jkf_0t}\,dt = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2}\cos(2\pi f_0 t)e^{-2\pi jkf_0t}\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}\left(e^{2\pi jf_0t} + e^{-2\pi jf_0t}\right)e^{-2\pi jkf_0t}\,dt$$

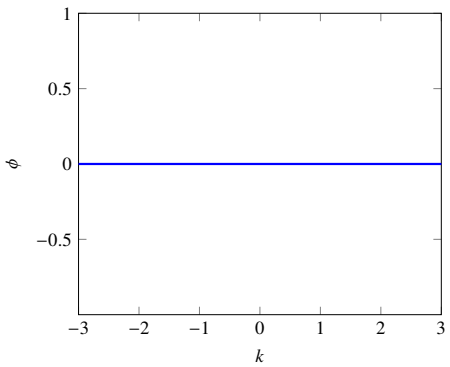
$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}\left(e^{2\pi jf_0t(1-k)} + e^{2\pi jf_0t(-1-k)}\right)dt = \frac{1}{T_0}\Bigg/_{-T_0/2}^{T_0/2}\left(\frac{e^{2\pi jf_0t(1-k)}}{2\pi jf_0(1-k)} + \frac{e^{2\pi jf_0t(-1-k)}}{2\pi jf_0(-1-k)}\right)$$

$$= \frac{e^{\pi j(1-k)} - e^{-\pi j(1-k)}}{2\pi j(1-k)} + \frac{e^{\pi j(-1-k)} - e^{-\pi j(-1-k)}}{2\pi j(-1-k)} = \frac{\sin(\pi(1-k))}{\pi(1-k)} + \frac{\sin(\pi(-1-k))}{\pi(-1-k)}$$

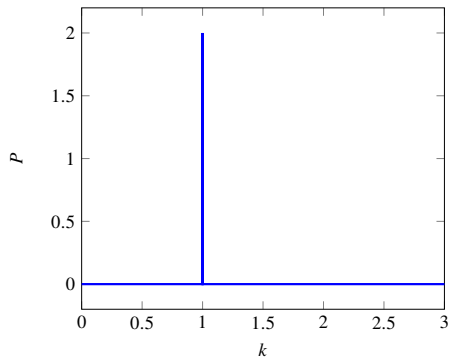
$$= \text{sinc}(1-k) + \text{sinc}(-1-k) = \text{sinc}(1-k) + \text{sinc}(1+k)$$

(c)





(d)



(e) Sinimuotoisen signaalin teho.

$$P = \frac{A^2}{2} = 2$$

3. (a) Signaalin energia

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = ((-1) - (-2)) \cdot (A/2)^2 + (1 - (-1)) \cdot A^2 + (2 - 1) \cdot (A/2)^2 = \frac{5}{2} A^2 = 1$$

$$A = \sqrt{\frac{2}{5}}$$

(b)

$$X(f) = \mathcal{F}\left\{\frac{A}{2}\mathrm{rect}\left(\frac{t}{4}\right) + \frac{A}{2}\mathrm{rect}\left(\frac{t}{2}\right)\right\} = \frac{A}{2}\mathcal{F}\left\{\mathrm{rect}\left(\frac{t}{4}\right)\right\} + \frac{A}{2}\mathcal{F}\left\{\mathrm{rect}\left(\frac{t}{2}\right)\right\} = 2A\mathrm{sinc}(4f) + A\mathrm{sinc}(2f)$$

$$|X(f)|^2 = (2A\mathrm{sinc}(4f) + A\mathrm{sinc}(2f))^2$$

(c) i.

$$H(f) = \mathcal{F}\{h(t)\} = \mathcal{F}\{\delta(t) + \delta(t-2)\} = 1 + e^{-4\pi jf}$$

ii.

$$Y(f) = \mathcal{F}\{y(t)\} = \mathcal{F}\{(x \otimes h)(t)\} = X(f)H(f) = A\left(1 + e^{-4\pi jf}\right)(2\mathrm{sinc}(4f) + \mathrm{sinc}(2f))$$

$$|Y(f)|^2 = A^2\left((1 + \cos(4\pi f))^2 + (\sin(-4\pi f))^2\right)(2\mathrm{sinc}(4f) + \mathrm{sinc}(2f))^2$$

$$= A^2\left(1 + 2\cos(4\pi f) + \cos^2(4\pi f) + \sin^2(4\pi f)\right)(2\mathrm{sinc}(4f) + \mathrm{sinc}(2f))^2$$

$$= 2A^2(1 + \cos(4\pi f))(2\mathrm{sinc}(4f) + \mathrm{sinc}(2f))^2$$