MS-E2140 Linear Programming
Final Exam **11.12.2017**

The exam is three hours long and consists of 4 exercises. The exam is graded on a scale of 0-25 points, and the points assigned to each question are indicated in parenthesis.

Problem 1

(a) Solve the following problem P by using the Dual Simplex algorithm. (6pt)

$$\begin{array}{lll} (P) & \text{Minimize} & 3x_1 + x_2 + x_3 \\ & \text{s.t.} & 2x_1 - 2x_2 + x_3 \leq 11 \\ & & 4x_1 - x_2 - 2x_3 \leq -3 \\ & & -2x_1 + x_3 \leq 1 \\ & & x_1, x_2, x_3 \geq 0. \end{array}$$

(b) Write the dual of P. (1pt)

Problem 2

Consider the following tableau for a minimization problem.

	$-z$	x_1	x_2	x_3	x_4	x_5
	-10	c	2	0	0	0
$x_3 =$	4	-1	a_1	1	0	0
$x_4 =$	1	a_2	-4	0	1	0
$x_5 =$	b	a_3	3	0	0	1

Give conditions on the unknown parameters b, c, a_1, a_2, a_3 that make the following statements true:

(a) The current (primal) basic solution is optimal. (1pt)

(b) The current basic solution is optimal and there are alternative (different) optimal solutions. (1pt)

(c) The LP is unbounded (i.e., the optimal cost is $-\infty$). (1pt)

(d) The current (primal) basic solution is not feasible. (1pt)

(e) The current (primal) basic solution is feasible but the objective value can be improved by bringing x_1 into the basis and removing x_4 from the basis. (1pt)

(f) The tableau is obtained at some iteration of the Dual Simplex method, and indicates that the dual problem is unbounded (the optimal dual cost is $+\infty$). (1pt)

Problem 3

Consider the following problem

$$
\begin{array}{lrl}
(P) & \text{Minimize} & -15x_1 - 8x_2 - 10x_3 - 12x_4 \\
& \text{s.t.} & x_1 + 2x_2 \qquad\quad + x_4 \leq 20 \\
& & x_1 + \ \ x_2 + x_3 + x_4 \leq 54 \\
& & 2x_1 \qquad\quad + x_3 + x_4 \leq 36 \\
& & x_1, x_2, x_3, x_4 \geq 0.
\end{array}
$$

and the corresponding optimal tableau where x_5, x_6 and x_7 are slack variables

	$-z$	x_1	x_2	x_3	x_4	x_5	x_6	x_7
	440	9	0	0	2	4	0	10
$x_2 =$	10	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0
$x_6 =$	8	$-\frac{3}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	1	-1
$x_3 =$	36	2	0	1	1	0	0	1

Let $\mathbf{x}^*$ be the basic optimal solution associated with this tableau and $\mathbf{B}$ the corresponding basis matrix. Answer the following questions (each of the questions is independent from the others).

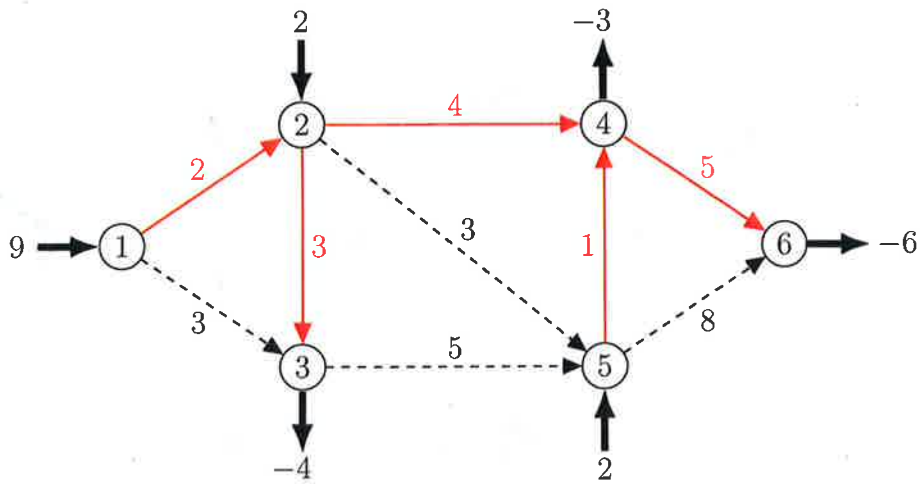
(a) Find $\mathbf{B}^{-1}$ and the optimal dual solution $\mathbf{p}^*$. Explain why $\mathbf{B}^{-1}$ and $\mathbf{p}^*$ can be obtained from the tableau. (2pt)

(b) If one of the right hand sides could be increased by an amount δ, which one is the most profitable to increase, and what is the range of values of δ that maintains $\mathbf{B}$ optimal? (2pt)

(c) Suppose that the cost of variable x_1 is changed to $-15 + \delta$. What is the range of values of δ that maintains $\mathbf{B}$ optimal? (2pt)

Problem 4

Consider the uncapacitated network flow problem defined on the directed graph below. The number next to each arc (i, j) is its cost c_{ij}.



Consider the tree T defined by the solid red arcs in the graph. Solve the problem by using the Network Simplex starting from the tree solution defined by T. (6pt)

(*i*) Indicate the set T and report the arc flows at each iteration

(*ii*) Report the dual variables at each iteration

(*iii*) Explain how the leaving and entering variables are selected

(*iv*) Explain how the flows are updated