ELEC-E-5440 Statistical Signal Processing. Final Exam December 14, 2017

1. Define or explain briefly the following concepts:

(a) Sufficient statistics

(b) Influence Function

(c) Bayes risk

(d) Divergence of Kalman Filter

(e) Consistency

(f) Array aperture and resolution

(f) Bias

(f) Invariance of ML estimator

2. Explain the principles of subspace based estimation of directions of arrival. Use MUSIC method to illustrate the principles. Discuss the the performance of the technique as well.

3. Let us have N i.i.d. observations from the *pdf:* $f_\theta(x) = e^{-(x-\theta)}$,
if $\theta < x < \infty$
and $f_\theta(x) = 0$ otherwise.

Find the Cramer-Rao lower bound for the variance of unbiased estimator of θ.

4. Suppose that Θ is a random parameter and given $\Theta = \theta$, the observation y have a density

$$f(y|\theta) = (\theta/2)e^{-\theta|y|},\ y \in R$$

Suppose further that Θ has prior density

$$f(\theta) = \begin{cases} 1/\theta, & 1 \leq \theta \leq e \\ 0 & \text{otherwise} \end{cases}$$

Find the MAP and Mean Square estimators of Θ based on observation.

Recall the Bayes rule:

$$f(\theta|\mathbf{y}) = \frac{f(\mathbf{y}|\theta)f(\theta)}{f(\mathbf{y})}$$

5 Let us have N independent and identically distributed (i.i.d.) observations $x_1, ..., x_N$ having the *pdf:*

$$f_\theta(x) = \theta^2 x e^{-\theta x},$$

where $\theta > 0$. Find the Maximum Likelihood estimator of θ. You want to study whether the obtained maximum likelihood estimator above is unbiased. Explain how you would do that.