

Final Exercise

Course: ELEC-E8103 Modelling, Estimation, and Dynamic Systems

# Final exercise 07.12.2017

The final exercise will start from 07.12.2017 09:00 to 12.12.2017 23:55. Please submit your solutions to the final exercise section of the course page in mycourses.aalto.fi before the deadline. The final exercise should be done individually, and NO discussion is allowed.

Submission of the final exercise

You should submit your solutions to the Final Exercise section of the Mycourses page: https://mycourses.aalto.fi/mod/assign/view.php?id=286148. Your submission should include a single zip file named as "surname\_studentNumber\_Final.zip", consisting of a pdf file named "surname\_studentNumber\_Final.pdf", the following MATLAB files: "problem1.m", "constant.m", "sys01.sid", "sys02.sid", and "sys03.sid" and a Simulink model "model.mdl".

The hard deadline for submission of the final exercise solutions is 12.12.2017 at 23:55.



#### Final Exercise

### 1. (35 points)

Consider the magnetic manipulation system illustrated in Fig.1. This system uses a mobile electromagnetic needle to trap magnetic particle. The goal of this exercise is to model the displacement of the magnetic particle to understand and control its motion.

When current is supplied to the needle, a magnetic force with the following form is generated:

$$\overrightarrow{F_m} = \frac{2}{3}\pi d_p^3 \rho_p M_p \frac{\beta M_n^2}{(4\beta\delta+1)^3} \overrightarrow{u}$$

Where  $\delta$  is the distance between the particle and the needle,  $\vec{u}$  is the unit vector going from the particle to the needle tip,  $M_p$  is the mass magnetization of the particle,  $M_n$  is the magnetization of the needle core (which is related to the supplied current),  $\beta$  a parameter related to the shape of the needle tip,  $d_p$  is the particle diameter and  $\rho_p$  the particle density.

The particle and the needle are placed in a small tank filled with liquid. The particle is subject to a drag force when a motion takes place inside the liquid (which is induce by the magnetic force and/or by the gravity). This force is opposed to the motion of the particle and can be expressed with the following equation:

$$\overrightarrow{F_d} = -\mathbf{k} \overrightarrow{v}_p$$

Where  $\vec{v}_p$  is the velocity of the particle and  $k = 3\pi v_f d_p$  the drag coefficient is function of fluid viscosity and particle diameter.

The particles are spherical, and their position is represented by the center of the sphere  $(x_p, y_p, z_p)$ . The needle is fixed, and its position is represented by the position of its tip  $(x_n, y_n, z_n)$ .



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Figure 1 – Magnetic manipulation using an EMN

a) Based on the description of this system, list the different forces that can affect the particle.

(2.5 points)

b) Using first principle, write the three differential equations (one for each axis) that describe the system

(9 points)

At the microscale, inertia is, most of the time, negligible because of the size of the objects. We assume that this hypothesis is true for the remaining of this exercise.

c) Rewrite the differential equations based on this hypothesis. You should obtain the following system of equation:

$$\begin{pmatrix} \dot{x_p} = \frac{\lambda}{\alpha} \left( \frac{x_n - x_p}{\delta(4\beta\delta + 1)^3} \right) \\ \dot{y_p} = \frac{\lambda}{\alpha} \left( \frac{y_n - y_p}{\delta(4\beta\delta + 1)^3} \right) \\ \dot{z_p} = \frac{\lambda}{\alpha} \left( \frac{z_n - z_p}{\delta(4\beta\delta + 1)^3} \right) - \frac{\gamma}{\alpha}$$



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Express  $\lambda$  as a function of  $d_p$ ,  $\rho_p$ ,  $M_p$ ,  $\beta$ ,  $M_n$ ,  $\alpha$  as a function of  $\nu_f$ ,  $d_p$  and  $\gamma$  as a function of  $d_p$ , g,  $\rho_p$ ,  $\rho_f$ . (10 points)

We assume that the needle and the particle are aligned in the z direction (no motion in x and y axis). We also assume that  $M_n$  is the input of the system and the position and velocity of the particle are the output. For the following questions, we consider these relation:

$$M_p = \theta_1 atan(\theta_2 B)$$

$$B=\frac{M_n}{4\beta\delta+1}$$

d) Based on these assumptions, design a Simulink model ("model.mdl") of this system. You should also save all the constant used in a script named "constant.m". Be sure to save your model with compatibility for Matlab R2017a.

(2.5 points)

e) Simulate your system for a duration of 0.5s and a step input of  $M_n = 8$ . Draw the output of your system in your report. What happen if you increase the simulation time? Is it physically possible?

(2.5 points)

We perform some measurement on the experimental setup (exp\_data.mat). Two parameters were monitored: the velocity of the particle and the distance between the needle and the particle. Unfortunately, the input  $M_n$  used for these experiments has not been saved. Based on the equation defined in question c, perform a regression and find an estimation for this missing parameter.

f) Write a Matlab script named "problem1.m" that compute and minimize the sum of squared error. What is the estimated value for the input?

(5points)

g) Compute the SSE and R<sup>2</sup> of your fit. (2.5 points)

h) Draw (in your report) the value of the SSE near the optimal value. (1 points)



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## Numerical values to use in this exercise:

 $\begin{aligned} d_p &= 10.\,10^{-6} \quad \rho_p = 3263 \quad \theta_1 = 2.\,05 \quad \theta_2 = 3.\,031 \quad \beta = 250000 \\ \nu_f &= 1.\,10^{-3} \quad \rho_f = 1000 \quad g = 9.\,81 \\ z_n &= 200.\,10^{-6} \quad z_p(0) = 50.\,10^{-6} \quad \dot{z}_p(0) = 0 \end{aligned}$ 



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### 2. (15 points)

The goal of this question is to identify dynamic systems using input-output data. Copy the files "runExam.m", and "modeldata.mat" in your MATLAB current folder. Run the "runExam.m" script.

The following text should appear in your command window.

### >> runExam

Please Enter the numeric part of your student number!:

Now, you should type your student number and press Enter. If your student number ends with an alphabetical letter, you should just type the numerical part of your student number, e.g. if your student number is 12345W, you should type 12345.

Then your data will be stored to MATLAB workspace. Data has been collected from three different dynamic systems. Input-output datasets are  $(u_1, y_1)$ ,  $(u_2, y_2)$ , and  $(u_3, y_3)$ . The sampling frequency for all of the datasets has been 1 Hz. Identify polynomial models of the mentioned systems using MATLAB System Identification Toolbox. You should explicitly select a model as your final answer for each input-output dataset. You are supposed to answer the following question for each system.

- a) What is your selected model structure? Include a snapshot of your final sid file in your report.
- b) Present the resulted plots and information related to validation procedure **only for your selected model structure**. (**Hint**: For instance residual analysis plots, poles and zeros plot, variance analysis information, etc.)
- c) What are the alternative model(s) for the data if you think there are any?

Save the final identification session for each dynamic system as "sys01.sid" for  $(u_1, y_1)$ , "sys02.sid" for  $(u_2, y_2)$ , and "sys03.sid" for  $(u_3, y_3)$ .

The points will be given based on the resulted model structures and their orders, as well as the identification path.

(5 points for identification of each system, 15 points in total)



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