Answer all five questions (in English, Finnish, or Swedish). Using a calculator is allowed, but all memory must be cleared!

1. Describe the field-oriented control system for permanent-magnet synchronous motors. Draw also the block diagram of the control system, label the signals in the diagram, and describe the tasks of the blocks.

   Solution:

   See lectures and readings.

2. Answer briefly to the following questions:
   (a) Why a speed reduction gear is often used in electric drives?
   (b) Why three-phase machines are preferred to single-phase AC machines?
   (c) How the physical size of the motor approximately depends on the rated values of the motor?

   Solution:

   See lectures, exercises, and readings.

3. A permanent-magnet synchronous motor is used to start a mechanical load. The total moment of inertia is 0.4 kgm^2 and the load torque is constant 10 Nm. The motor is fed from an inverter using the control principle $i_\mathrm{d} = 0$. The rated power of the motor is 3.75 kW and the rated speed is 2 400 r/min. The current limit of the inverter has been set to twice the rated current ot the motor. How long does it take to accelerate the motor and its connected mechanical load from zero to the speed of 2 000 r/min?

   Solution:

   The rated angular rotor speed is

$$\omega_\mathrm{N} = 2\pi n_\mathrm{N} = 2\pi \cdot \frac{2\,400\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 251.3\ \mathrm{rad/s}$$

   The rated torque of the motor is

$$T_\mathrm{N} = \frac{P_\mathrm{N}}{\omega_\mathrm{N}} = \frac{3\,750\ \mathrm{W}}{251.3\ \mathrm{rad/s}} = 14.9\ \mathrm{Nm}$$

   During the acceleration, the stator current is twice its rated value. Since the torque of the motor is proportional to the stator current ($i_\mathrm{d} = 0$), the motor produces twice the rated torque $T_\mathrm{M} = 2T_\mathrm{N}$. The final angular rotor speed after the acceleration is

$$\omega_\mathrm{M} = 2\pi \cdot \frac{2\,000\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 209.4\ \mathrm{rad/s}$$

   Hence, the acceleration time is

$$\Delta t = J\frac{\Delta\omega_\mathrm{M}}{T_\mathrm{M} - T_\mathrm{L}} = 0.4\ \mathrm{kgm}^2 \cdot \frac{209.4\ \mathrm{rad/s}}{2 \cdot 14.9\ \mathrm{Nm} - 10\ \mathrm{Nm}} = 4.2\ \mathrm{s}$$

4. A DC motor with a separately excited field winding is considered. The rated armature voltage is $U_\mathrm{N} = 500$ V, rated torque $T_\mathrm{N} = 220$ Nm, rated speed $n_\mathrm{N} = 1\,600$ r/min, and maximum speed $n_\mathrm{max} = 3\,200$ r/min. The losses are omitted.
   (a) The flux factor k_f is kept constant at its rated value. When the armature voltage is varied from 0 to U_N, the speed varies from 0 to n_N. Determine the rated armature current I_N.
   (b) A load is to be driven in the speed range from n_N to n_max by weakening the flux factor while the armature voltage is kept constant at U_N. Determine the torque available at maximum speed, if the rated armature current I_N is not exceeded.
   (c) Sketch the armature voltage U_a, flux factor k_f, torque T_M, and mechanical power P_M as a function of the speed, when the armature current is kept at I_N. Clearly label axes of your graph.

Solution:

The losses are omitted, i.e., $R_\mathrm{a} = 0$ holds. Hence, the steady-state equations of the DC motor are

$$U_\mathrm{a} = k_\mathrm{f}\omega_\mathrm{M} \qquad T_\mathrm{M} = k_\mathrm{f} I_\mathrm{a} \qquad P_\mathrm{M} = T_\mathrm{M}\omega_\mathrm{M} = U_\mathrm{a} I_\mathrm{a}$$

(a) Let us first calculate the rated rotor speed in radians per second:

$$\omega_\mathrm{N} = 2\pi n_\mathrm{N} = 2\pi \cdot \frac{1600\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 167.6\ \mathrm{rad/s}$$

The rated flux factor is

$$k_\mathrm{fN} = \frac{U_\mathrm{N}}{\omega_\mathrm{N}} = \frac{500\ \mathrm{V}}{167.6\ \mathrm{rad/s}} = 2.98\ \mathrm{Vs}$$

The rated armature current is

$$I_\mathrm{N} = \frac{T_\mathrm{N}}{k_\mathrm{fN}} = \frac{220\ \mathrm{Nm}}{2.98\ \mathrm{Vs}} = 73.8\ \mathrm{A}$$

(b) The maximum rotor speed in radians per second is

$$\omega_\mathrm{max} = 2\pi n_\mathrm{max} = 2\pi \cdot \frac{3200\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 335.1\ \mathrm{rad/s}$$

The flux factor at the maximum speed is

$$k_\mathrm{f} = \frac{U_\mathrm{N}}{\omega_\mathrm{max}} = \frac{500\ \mathrm{V}}{335.1\ \mathrm{rad/s}} = 1.49\ \mathrm{Vs}$$

The torque at the maximum speed is

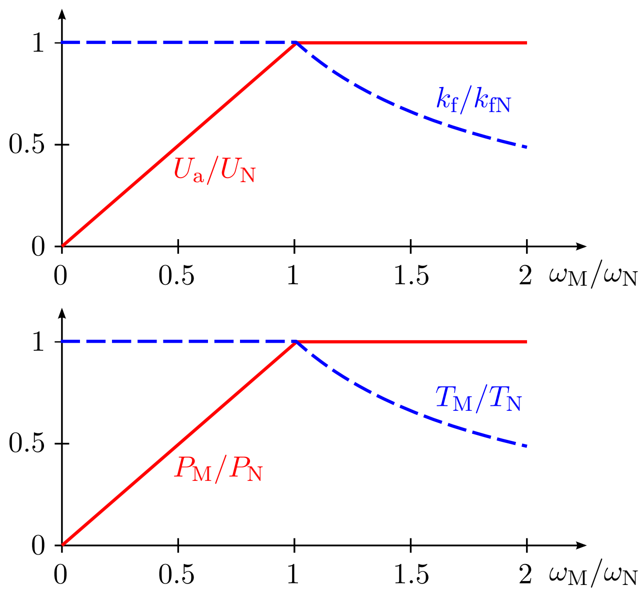
$$T_\mathrm{M} = k_\mathrm{f} I_\mathrm{N} = 1.49\ \mathrm{Vs} \cdot 73.8\ \mathrm{A} = 110\ \mathrm{Nm}$$

The same result could be obtained as $T_\mathrm{M} = (n_\mathrm{N}/n_\mathrm{max})T_\mathrm{N}$, i.e. the torque reduces inversely proportionally to the speed in the field-weakening region.

(c) The requested characteristics are shown in the figure below.

Based on $U_\mathrm{a} = k_\mathrm{f}\omega_\mathrm{M}$, the armature voltage increases linearly with the rotor speed until the rated (maximum) voltage U_N is reached at the rated speed. In order to reach higher speeds, the flux factor k_f has to be reduced inversely proportionally to the speed.

Since $I_\mathrm{a} = I_\mathrm{N}$ is constant, the torque $T_\mathrm{M} = k_\mathrm{f}I_\mathrm{a}$ follows the characteristics of the flux factor k_f. Based on $P_\mathrm{M} = T_\mathrm{M}\omega_\mathrm{M}$, the mechanical power $P_\mathrm{M} = T_\mathrm{M}\omega_\mathrm{M}$ increases linearly with the speed until the rated speed and remains constant at speeds higher than the rated speed. It is important to notice the same mechanical power is obtained also using the electrical quantities, $P_\mathrm{M} = U_\mathrm{a}I_\mathrm{a}$, since the losses are omitted.



Please turn the page for Problem 5.

5. Consider an inverter-fed permanent-magnet synchronous motor, whose rated speed is 2 400 r/min and number of pole pairs is $p = 4$. The motor parameters are determined using the following three tests:

   (a) The rotor speed is zero. The constant current vector $\underline{i}_\mathrm{s} = i_\mathrm{d} = 15.0$ A is fed into the stator winding by means of closed-loop current control. In this steady-state condition, the voltage vector is $\underline{u}_\mathrm{s} = u_\mathrm{d} = 7.5$ V according to the inverter control algorithm. Determine the stator resistance R_s.

   (b) The rotor speed is zero also in this test. The inverter produces a pulsating sinusoidal voltage into the d-axis, while the q-axis voltage is kept zero, i.e.,

   $$\underline{u}_\mathrm{s}(t) = u_\mathrm{d}(t) = U \sin(\omega_\mathrm{c} t)$$

   where the amplitude is $U = 50$ V and the angular frequency is $\omega_\mathrm{c} = 2\pi \cdot 200$ rad/s. The measured d-axis current response is

   $$\underline{i}_\mathrm{s}(t) = i_\mathrm{d}(t) = I \sin(\omega_\mathrm{c} t + \phi)$$

   where the amplitude is $I = 8$ A and the phase is approximately $\phi = -\pi/2$. Determine the stator inductance L_s (assume $R_\mathrm{s} = 0$ due to the high frequency). What is the torque produced by the motor during this test?

   (c) The motor is controlled to rotate at the speed of 1 200 r/min in a no-load condition. The line-to-line rms voltage of 190.8 V is supplied by the inverter. Determine the PM flux linkage ψ_f.

Solution:

In rotor coordinates, the stator voltage and the flux linkage are

$$\underline{u}_\mathrm{s} = R_\mathrm{s}\underline{i}_\mathrm{s} + \frac{\mathrm{d}\underline{\psi}_\mathrm{s}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{m}\underline{\psi}_\mathrm{s} \tag{1}$$

$$\underline{\psi}_\mathrm{s} = L_\mathrm{s}\underline{i}_\mathrm{s} + \psi_\mathrm{f} \tag{2}$$

respectively. The parameters R_s, L_s, and ψ_f are determined using the three tests.

(a) This test is carried out in the steady state at zero speed, i.e., $\mathrm{d}/\mathrm{d}t = 0$ and $\omega_\mathrm{m} = 0$ hold. Hence, the stator resistance is simply

$$R_\mathrm{s} = \underline{u}_\mathrm{s}/\underline{i}_\mathrm{s} = 7.5 \text{ V}/15.0 \text{ A} = 0.5\ \Omega$$

(b) The second test is also carried out at zero speed, $\omega_\mathrm{m} = 0$. Inserting (2) into (1) and taking the real part yields the d-axis voltage

$$u_\mathrm{d} = R_\mathrm{s} i_\mathrm{d} + L_\mathrm{s}\frac{\mathrm{d}i_\mathrm{d}}{\mathrm{d}t} \tag{3}$$

where $\mathrm{d}\psi_\mathrm{f}/\mathrm{d}t = 0$ since ψ_f is constant. Assuming $R_\mathrm{s} = 0$, the stator inductance is obtained directly by representing sinusoidal functions using the phasors:

$$L_\mathrm{s} = \frac{U}{\omega_\mathrm{c} I} = \frac{50 \text{ V}}{2\pi \cdot 200 \text{ rad/s} \cdot 8 \text{ A}} = 5 \text{ mH}$$

Alternatively, the time-domain voltage equation (3) can be used with

$$i_\mathrm{d}(t) = I\sin(\omega_\mathrm{c}t - \pi/2) = -I\cos(\omega_\mathrm{c}t) \qquad \Rightarrow \qquad \frac{\mathrm{d}i_\mathrm{d}(t)}{\mathrm{d}t} = \omega_\mathrm{c}I\sin(\omega_\mathrm{c}t)$$

which naturally leads to the same result. The torque expression is

$$T_\mathrm{M} = \frac{3p}{2}\,\mathrm{Im}\left\{\underline{i}_\mathrm{s}\underline{\psi}_\mathrm{s}^*\right\} = \frac{3p}{2}\psi_\mathrm{f}i_\mathrm{q}$$

Since the q-axis current is zero during the test, no torque is produced.

(c) During this test, the electrical angular speed of the rotor is

$$\omega_\mathrm{m} = p\omega_\mathrm{M} = 4\cdot 2\pi\cdot\frac{1\,200\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 2\pi\cdot 80\ \mathrm{rad/s}$$

At this speed, the line-to-neutral voltage $e_\mathrm{s} = \sqrt{2/3}\cdot 190.8\ \mathrm{V} = 155.8\ \mathrm{V}$ is induced in the stator winding. Hence, the PM flux linkage is

$$\psi_\mathrm{f} = \frac{e_\mathrm{s}}{\omega_\mathrm{m}} = \frac{155.8\ \mathrm{V}}{2\pi\cdot 80\ \mathrm{rad/s}} = 0.31\ \mathrm{Vs}$$