ELEC-E3151 - Mathematical Computing

Non-obligatory course exam - MATLAB part

12th of December 2017, 2 pm. @ OIH

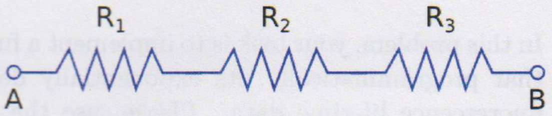
In this exam, you are required to answer only two of the following problems. If you choose to answer all three problems, the problem that is worth the least points will not be taken into account in the exam grading. All three exercises have a maximum score of 10 points.

The description of each individual problem contains a section called "Submission format" that states how you are supposed to submit your answer to that specific problem. Read these instructions very carefully. In general, you are supposed to upload your answers in separate .m files, one for each problem.

Exercise 1 – Linear programming (10 pts)

Use the command *linprog* to solve the following optimization problem:

Minimize the power dissipated by the set of resistors depicted in the picture, by choosing the appropriate resistors.



The constraints are:

- R_1 must be at least 10 ohms
- $R_2 + R_3$ must be at least 10 ohms and at most 100 ohms
- $R_2 - R_1$ must be at least 20 ohms

The current flowing from terminal A to terminal B is 0.1 A. If you remember your electronics classes, you can already calculate the right answer and compare it with the result of your script.

Submission format:

- Use the ready-made template (MC2017_exam_optimization_XXXXXX.m) for your submission.
- Name your submission file as MC2017_exam_optimization_STUDENTNUMBER.m.

Exercise 2 – Fourier analysis (10 pts)

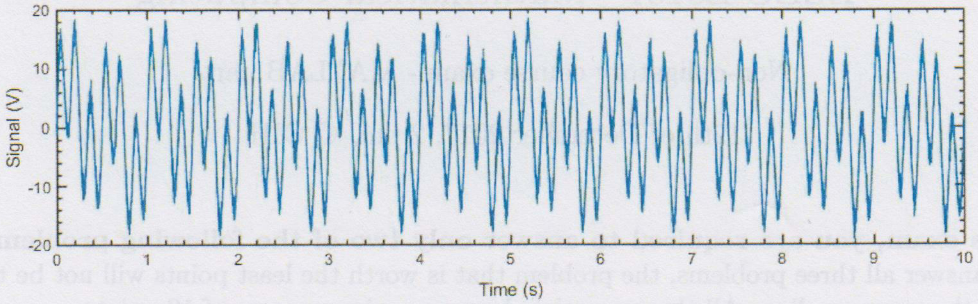
You are provided with data of a noisy oscillating signal (MC2017_exam_fourier_data.txt). Write a script that calculates the Fourier transform of the data and plot the power spectrum of the signal. Identify the main frequency components composing the signal. Furthermore, observe the high frequency noise spectrum and study the frequency dependence of the overlaid noise.

In the beginning of your script, answer also the following questions about the power spectrum (use comment lines):

1. Which main frequency components is the signal composed of?
2. Which of the following types of noise spectrum is present in the data (choose one):
   (a) White noise (noise power spectrum constant, $\propto f^0$)

(b) Pink noise (noise spectrum $\propto f^{-1}$)

(c) Blue noise (noise spectrum $\propto f^{1}$)

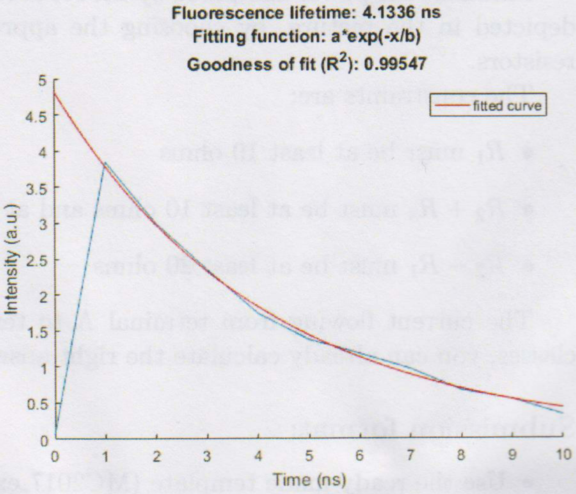


Submission format:

- Return your script as a single .m file.
- Name your submission file as MC2017_exam_fourier_STUDENTNUMBER.m.

Exercise 3 – Programmatic fitting (10 pts)

In this problem, your task is to implement a function that programmatically fits exponentially decaying fluorescence lifetime data. Please, use the ready-made template for your solution (see below). Make changes only to the function, i.e., leave the rest of the script as it is. The output of the final script should look like the figure on the right.



Fluorescence lifetime refers to the average time a molecule stays in its excited state before emitting a photon. In general, the fluorescence intensity of a collection of molecules decays exponentially with time. In other words, the fluorescence intensity I can be written as

$$I = I_0 \exp(-t/\tau),$$

where I_0 is proportional to the overall fluorescence intensity and τ is the fluorescence lifetime. Both I_0 and τ are always non-negative.

For common fluorophores, excited state lifetimes are within the range of 0.5 to 20 ns. The fluorescence lifetime is an important parameter for practical applications of fluorescence, such as fluorescence resonance energy transfer and fluorescence-lifetime imaging microscopy.

Submission format:

- Use the ready-made template (MC2017_exam_fitting_XXXXXX.m) for your submission.
- Name your submission file as MC2017_exam_fitting_STUDENTNUMBER.m.

Mathematica part of Exam 12th of Nov.

Solve two out of the three problems below. Use the template notebook from MyCourses when creating your solutions. Write your Mathematica code in systematic order with explanations as comments. Unclear program code will automatically produce less points.

1. Consider the following pair of nonlinear equations

$$\begin{cases} x^y + y^x = 2, & x,\, y > 0 \\ x^x + y^y = 3, & x,\, y > 0 \end{cases}$$

Both equations define a curve in their domain of definition.

a) Solve numerically the pair of equations in order to find the crossings of the curves

b) Illustrate the solution by creating a plot showing both curves and highlighting the solutions for x and y pairs by a marker e.g. a point

2. Consider the differential equation (DE) with boundary conditions

$$y''(x) + 3y'(x) - 4y(x) = 0, \qquad y(0) = 2 \wedge y'(0) = 4$$

a) Solve it analytically and check by substitution that a proper solution has been found

b) Solve the same DE also numerically over the interval $0 \leq x \leq 4$ resulting an approximation $y_n(x) \approx y(x)$ in terms of an interpolation function. Set the options *InterpolationOrder* to *All* and *WorkingPrecision* to 10 in the solver. Create a plot illustrating the interpolating function over the defined interval.

c) Determine the (integer) value for WorkingPrecision which forces the integral of the error smaller than the given limit below

$$\int_0^4 |y(x) - y_n(x)|\, dx < 10^{-6}$$

3. Consider the tangent function as the ratio of sine and cosine functions

$$\tan x = \frac{\sin x}{\cos x}$$

a) Develop a rational approximation for it by approximating separately the numerator and denominator by a Taylor's series with only two terms. The Taylor's series should be developed at the origin. Plot $\tan x$ and its approximation over the interval $-1.5 < x < 1.5$ and specify the curves in the plot

b) Derive an alternative rational approximation for the tangent function by first taking 10 equidistant data samples of $\tan x$ over the interval $-1.5 < x < 1.5$ and then fitting a mathematical model

$$f(x) = \frac{ax + bx^3}{c + dx^2}$$

to the data. What are the values of the model parameters a, b, c and d?

c) Plot $\tan x$ and the two approximations above in the same plot and specify the curves in the figure.