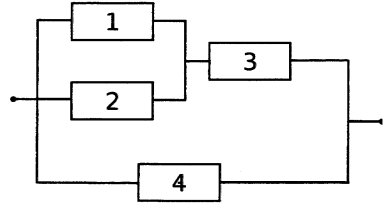


Please answer to all five (5) questions. Use of calculators/additional material is not allowed in the exam.

1. Consider a pure single server queueing system with average service rate of 2.5 customers/s. New customers arrive at rate 2.0 customers/s. The average total delay is 3.0 s (including both the waiting time and the service time).
  - (a) What is the average number of customers in the whole system?
  - (b) What is the average number of waiting customers?
  - (c) What is the average number of customers in service?
  - (d) What is the average number of departing customers during an interval of length 10 s?
2. Consider a queueing system with two parallel servers. The service times are independent and identically distributed following the  $\text{Exp}(1/2)$  distribution. The system is empty at time 0. Two new customers arrive at times 1 and 2, respectively, and no other customers enter the system. In addition, it is known that both customers are still in the system at time 3. Let  $Z_1$  denote the time at which the customer with the shorter service time leaves the system. Correspondingly, let  $Z_2$  denote the time at which the customer with the longer service time departs. Thus,  $Z_2 > Z_1 > 3$ . Determine the mean values  $E[Z_1]$  and  $E[Z_2]$ .
3. Consider elastic data traffic carried by a 10-Mbps link in a packet switched network. Use a pure sharing system model with a single server. New flows arrive according to a Poisson process at rate 9 flows per second, and the sizes of files to be transferred are independently and exponentially distributed with mean 1 Mbit. Let  $X(t)$  denote the number of ongoing flows at time  $t$ .
  - (a) What is the traffic load?
  - (b) Derive the equilibrium distribution of  $X(t)$ .
  - (c) What is the throughput of a flow?
4. Consider the M/M/3/4 model where customers arrive at rate  $\lambda$  customers per time unit and the mean service time is  $1/\mu$  time units. Let  $X(t)$  denote the number of customers in the system at time  $t$ .
  - (a) Draw the state transition diagram of the Markov process  $X(t)$ .
  - (b) Derive the equilibrium distribution of  $X(t)$ .
  - (c) Assuming that  $\lambda = \mu$ , what is the probability that the arriving customer can enter service directly without any waiting?

Last question on the other side of the paper

5. (a) Determine the structure function  $\phi(\mathbf{x})$  of the system of independent components in the reliability block diagram below.



- (b) If the components in above diagram are repairable, what is the availability of the above system? The mean time to failure of each component  $i$ ,  $MTTF_i$ , are  $MTTF_1 = MTTF_2 = 2$  hours and  $MTTF_4 = 1$  hour. The mean down time of component  $i$ ,  $MDT_i$ , for components 1, 2 and 4 is one hour. The component 3 cannot break down so the availability for component 3 is 1.