MEC-E1040 Dynamics of Structures — Luc St-Pierre

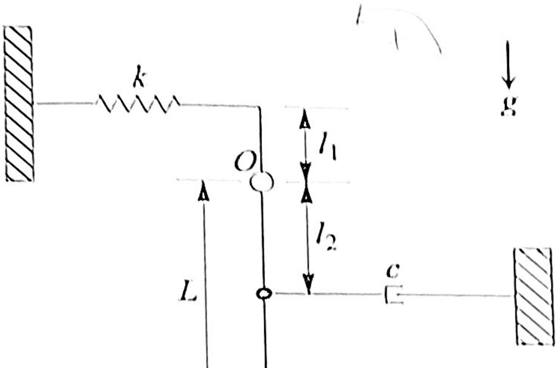
Exam — Dec 14, 2017, 1-5pm

A? Problem 1 (15pts)

Consider the pendulum illustrated below, which is pivoted at point O. Assume that rotations are small and that the mass of the rod is negligible.

1. Find the equation of motion for the system. Recall that the moment of inertia of the pendulum is mL^2.

2. Based on the equation of motion, find the natural angular frequency ω_n and the damping ratio ζ of the system.



A? Problem 2 (25pts)

An atomic force microscope has a mass $m = 100$ kg and its protective casing is made of a visco-elastic material with a damping coefficient $c = 50$ kg/s. Researchers have found that the metro is making the floor vibrate with an amplitude $Y = 3$ cm and an angular frequency $\omega_b = 3$ rad/s.

1. Calculate the vibration amplitude considering that the mounting system has a stiffness $k = 1$ kN/m.

2. Design a new mounting system (find a new value of k) that would ensure that the vibration amplitude of the microscope do not exceed 2.25 cm.

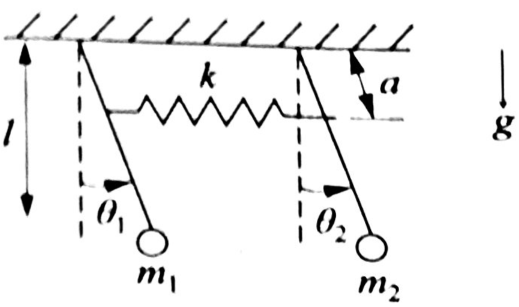
A? Problem 3 (10pts)

A rotating machine has a small imbalance. It has been observed that at resonance ($r = 1$), the vibration amplitude is 10 mm. As the rotating speed is increased several decades past resonance, the amplitude of displacement remains fixed at 1 mm. Estimate the damping ratio ζ of the system.

A? Problem 4 (25pts)

Consider the system below with two pendulums of length l connected by a spring. Assume that rotations are small and that the rods have a negligible mass.

1. Use Lagrange equation to derive the equations of motion. Express your result in a matrix form.
2. Compute the natural angular frequencies of the system and the mode shapes, provided that $k = 20\,\text{N/m}$, $l = 0.5\,\text{m}$, $a = 0.1\,\text{m}$ and $m_1 = m_2 = 10\,\text{kg}$.



A? Problem 5 (25pts)

A machine is designed with a steady-state operating speed between 2000 and 4000 rpm. Unfortunately, due to an imbalance in the machine, a large violent vibration occurs at 3000 rpm. An initial absorber is installed with a mass $m_a = 2\,\text{kg}$ tuned to 3000 rpm. This, however, causes the natural frequencies of the system to occur at 2400 and 3750 rpm.

1. Find the mass m and stiffness k of the machine. Assume that the system is undamped.
2. Redesign the absorber (find new values for m_a and k_a) so that the natural frequencies of the system are 2000 and 4000 rpm, rendering the system safe for operation. Again, assume that the system is undamped.