ELEC-E8403 Converter Techniques **Exam 6.4.2017**

Question 1

A phase-controlled six-pulse rectifier, fed from a 230 V, 50 Hz line, operates with a firing angle of 60°. The ac source inductance is such that the DC-output voltage of the rectifier is reduced 15 % in comparison to that with an ideal source. Assuming the continuous conduction mode, and calculate the overlap angle and DC-output voltage of the rectifier.

Question 2

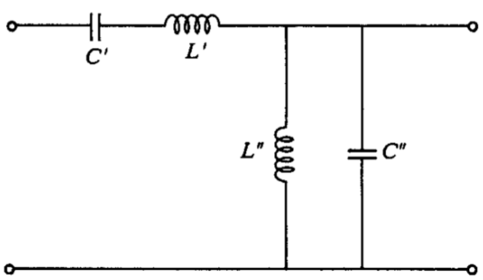
A three -phase current-type or current-source rectifier is used for ac-dc conversion. Modulation of the converter is based on space vectors. Derive equations for the durations of different space vectors used in the modulation. What is the maximum achievable ac current reference if the dc current value is 100 A.

Question 3

Pulse width modulation (PWM) is used to control the output voltage of a voltage sourced three-phase inverter to achieve a constant flux in the load ac motor. The DC bus voltage is 540 V, and the nominal line-to-line voltage and frequency of the three-phase motor are 400 V and 50 Hz respectively. The voltages of the motor form a symmetrical three-phase system. Why is it not possible to change the pulse pattern, three pulses per half-cycle, to one pulse per half-cycle pulse pattern at the frequency of $f = 50$ Hz? At what frequency this would be possible without a sudden change in the voltage?

Question 4

The filter shown below is tuned for 50 Hz. The input voltage to the filter is 50 Hz square-wave where the peak value of the fundamental component is 230 V. The peak value of 50 Hz current flowing through the filter is 10 A and the maximum allowed voltage of C′ is 230 V. The third harmonic component in the input voltage is one third of the fundamental component and in the output the allowed value is 2 % of the fundamental. Calculate the component values of the filter by assuming that the load has no effect on the filtering.



Question 5

Compare phase control and pulse width modulation with each other's. What are the advantages and disadvantages of these modulation methods. Give an example of converters using these methods.

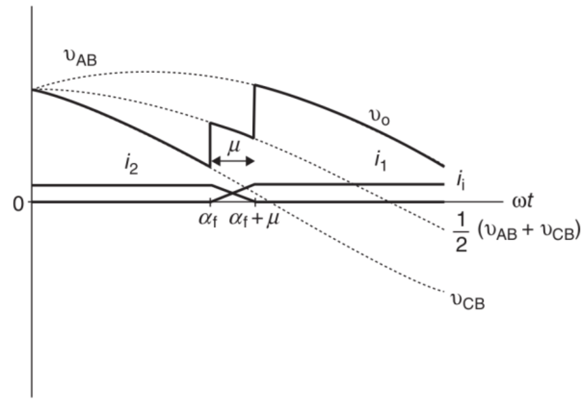
ELEC-E8403 Converter Techniques **Exam 6.4.2017**

Question 1

A 6-pulse thyristor rectifier operates with a firing angle of 60º. If the commutation is neglected, i.e. there is no inductance on the supplying ac system, dc voltage can be calculated from

$$V_{\mathrm{o,dc(C)}} = \frac{1}{\frac{\pi}{3}} \int_{\alpha_f}^{\alpha_f+\frac{\pi}{3}} V_{\mathrm{LL,p}} \sin\left(\omega t + \frac{\pi}{3}\right) d\omega t = \frac{3}{\pi} V_{\mathrm{LL,p}} \cos(\alpha_f)$$

which is Eq. (4.41) in the textbook. With the given values, the dc voltage is 269 V.



Waveforms of voltage and current in a phase-controlled six-pulse rectifier during commutation

When the commutation is taken into account, dc voltage drops and it can be calculated from

$$\Delta V_{o,DC} = \frac{3}{\pi} X_s I_{o,DC}$$

which is Eq. (4.64) in the textbook. In the given case the voltage drop is 15 %, i.e. 40,3 V and the output voltage of the converter is 228,7 V.

The overlap (or commutation) angle can be calculated from equation

$$\frac{V_{\mathrm{LL,p}}}{2X_s}[\cos(\alpha_f) - \cos(\alpha_f + \mu)] = I_{\mathrm{o,dc}}$$

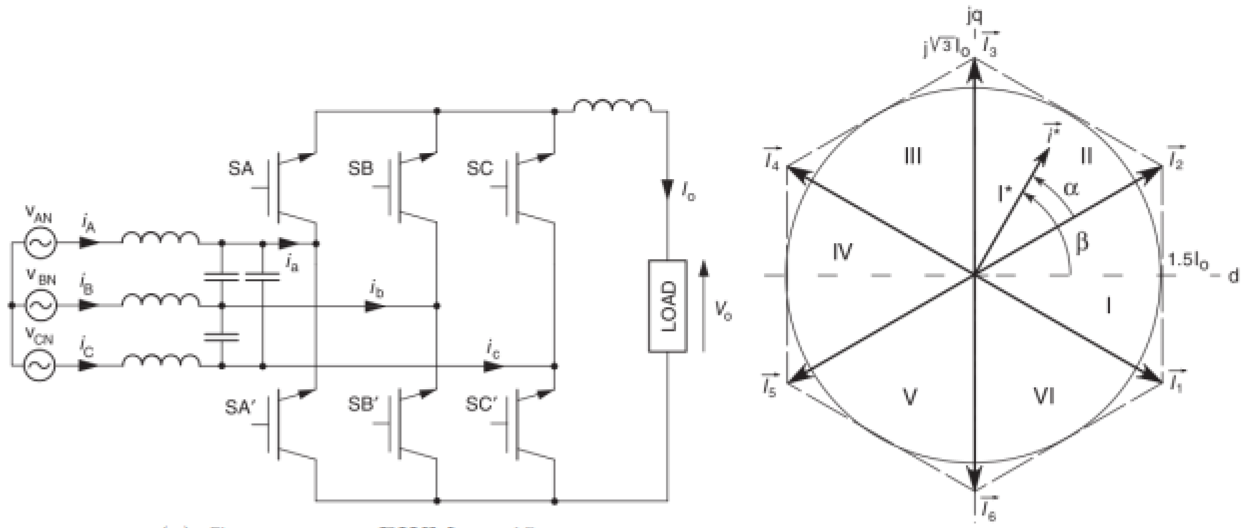
which has been derived in the textbook, Eq. (4.62). The first term $\frac{V_{\mathrm{LL,p}}}{2X_s}$ can be considered as the short circuit current of the supplying ac system. Equation for the commutation angle is thus

$$\mu = \left| cos^{-1}\left(cos(\alpha_f) - 2\frac{X_s I_{o,DC}}{V_{LL,p}} \right) - \alpha_f \right|$$

and with numerical values the result is 9,5º.

In many answers 230 V was considered as the line-to-line voltage, though it is line voltage. In this case the ideal dc voltage is 155,3 V, voltage drop 23,3 V, output voltage 132 V and commutation angle still 9,5º. Because it was not stated clearly in the question paper whether 230 is line-to-line or line voltage, both of the answers were considered correct.

Question 2



(a) Current-type PWM rectifier.

(b) Reference current vector in the vector space of input currents of the current-type PWM-rectifier

Figure 1: Current-type PWM rectifier.[1]

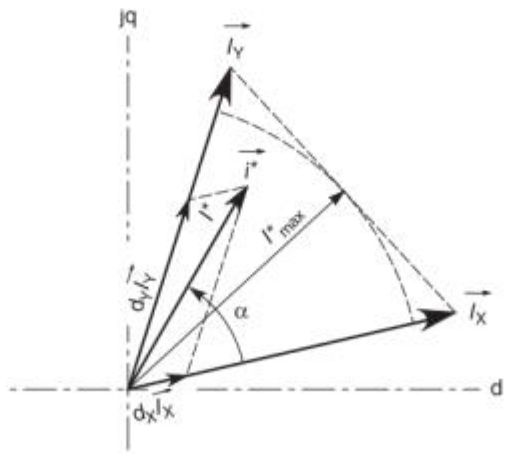


Figure 2: Synthesis of a rotating space vector $\vec{i^*}$ from stationary vectors $\vec{I_X}$ and $\vec{I_Y}$ [1].

The maximum available current reference is at a = pi/6 and therefore

$$I^*_{\max} = \cos(\frac{\pi}{6})|\vec{I_X}| = \cos(\frac{\pi}{6})|\vec{I_Y}| \quad \text{and} \quad I^*_{\max} = \frac{\sqrt{3}}{2}|\vec{I_X}| = \frac{\sqrt{3}}{2}|\vec{I_Y}|$$

The reference is created by using the two non-zero current vectors and the zero vector and the zero current vector. Thus the durations can be calculated from

$$\vec{i^*} = d_X\vec{I_X} + d_Y\vec{I_Y} \quad \text{and} \quad \vec{i^*} = I^*e^{j\alpha} = d_X I_X e^{j0} + d_Y I_X e^{j\frac{\pi}{3}}$$

By using Euler equation and treating real and imaginary part separately we end up to

$$I^*cos(\alpha) = d_X I_X + \frac{1}{2}d_Y I_X \qquad I^*sin(\alpha) = \frac{\sqrt{3}}{2}d_Y I_X$$

By defining modulation index

$$m = \frac{I^*}{I^*_{max}} = \frac{2I^*}{\sqrt{3}\vec{I_X}}$$

We can obtain the duration of the Y-vector to be $d_Y = msin(\alpha)$
And for X-vector

$$d_x = \frac{I^*}{I_X}\left(\cos(\alpha) - \frac{1}{\sqrt{3}}\sin(\alpha)\right) = \frac{2I^*}{I_X\sqrt{3}}\left(\frac{\sqrt{3}}{2}\cos(\alpha) - \frac{1}{2}\sin(\alpha)\right) = m\left(\frac{\sqrt{3}}{2}\cos(\alpha) - \frac{1}{2}\sin(\alpha)\right)$$
$$= m\, sin\left(\frac{\pi}{3} - \alpha\right)$$

When the switching cycle T_{sw} is divided between the previous vectors, time durations are

$$T_X = mT_{sw}sin(\frac{\pi}{3} - \alpha)$$

$$T_Y = mT_{sw}sin(\alpha)$$

$$T_Z = T_{sw} - T_X - T_Y$$

It was further asked, what is the maximum available current reference. The definition of space vector is from Eq (4.72) in the textbook

$$\vec{F}_s = \mathcal{F}_{as} + \mathcal{F}_{bs}e^{j120°} + \mathcal{F}_{bs}e^{j240°}$$

And it can be also expressed in matrix form as

$$\vec{i} = \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$

When e.g. current in phases A, B and C are I_o, $-I_o$ and 0, respectively, the corresponding space vector of input current is

$$\vec{I_1} = \frac{3}{2}I_o - j\frac{\sqrt{3}}{2}I_o$$

Which means that the length of the vector is equal to $\sqrt{3}\, I_o$. It was already shown that

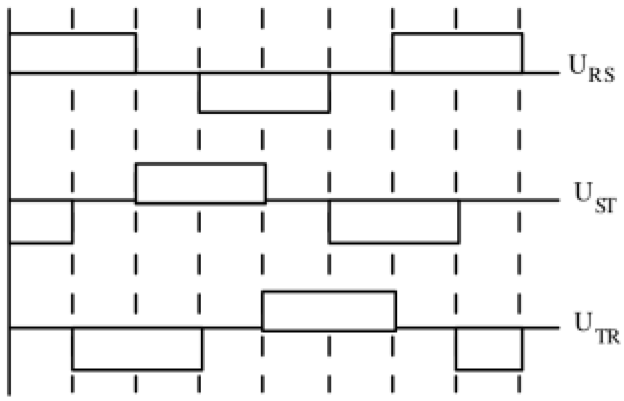
$$I^*_{max} = \frac{\sqrt{3}}{2}|\vec{I_X}| = \frac{\sqrt{3}}{2}|\vec{I_Y}|$$

Therefore, the maximum available current reference is I_o*3/2 and with 100 A dc current the result is thus 150 A.

In some answer's the space vector has been defined so that it is scaled with 2/3, which is often used in the literature too. In this case the above derived time durations need to be scaled by 2/√3 and the length of the space-vector is 2/√3. The maximum available current reference is then √3/2*100 A ≈ 87 A.

Question 3

Line-to-line voltages with one pulse per half cycle are shown below. They are created so that every phase is connected half of the cycle to positive and the other half cycle to the negative dc bus. Between the phases there is 120 degrees phase shift. Because of this, it is impossible to obtain wider pulses for the line-to-line voltages. On the other hand, it is also impossible to reduce the widths of the pulses. This would require that there are periods of time when a phase is not connected at all to the dc bus. As the connected load is always inductive, there needs to be a bath for the current to flow. Even if the IGBTs of the inverter are turned of the antiparallel connected diodes are conducting and connecting the phase to the dc bus.



The rms value of the fundamental, U1, is obtain from the Fourier coefficients,

$$U_1 = \frac{B_n}{\sqrt{2}} = \frac{2}{\pi\sqrt{2}} \int_{\pi/6}^{5\pi/6} E_d \sin\theta d\theta = \frac{\sqrt{6}}{\pi} E_d \approx 421\text{V}$$

The value of U_1 is 21V higher than U_N . The scheme ”1 pulse per half-cycle” cannot be modified for a lower value of U_1 due to the fact described above.

In the constant flux region, i.e. frequencies below the nominal frequency the ratio

$$\frac{U_1}{f} = \frac{U_N}{f_N} = \text{constant}$$

holds. Therefore, the above shown one pulse pattern can be used at frequencies above

$$f = \frac{U_1}{U_N} f_N \approx 52,62\text{Hz}$$

At lower frequencies some other pulse pattern must be used, e.g. three pulses per half cycle.

Question 4
For the series filter

$$Z'_{(n)} = jL'n\omega - j\frac{1}{C'n\omega}$$

and it is tuned for the fundamental component so that impedance is zero, i.e.

$$L'\omega = \frac{1}{C'\omega} = X'$$

The allowed voltage over the series capacitor with 10 A current is 230 V. From this we can calculate the parameters of the series filter

$$X' = \frac{1}{C'd\omega} = \frac{U_{C'}}{I_1} = 23\Omega = L'\omega$$

and we get $C' \approx 138$ μF and $L' \approx 73$ mH. In order to calculate the components of the parallel filter we need to calculate the damping. Admittance of the parallel filter is

$$Y''_{(n)} = jC''n\omega - j\frac{1}{L''n\omega} \quad \text{and} \quad C''\omega = \frac{1}{L''\omega} = Y''$$

Filtering of the filter is calculated from the voltage ratio

$$f_{(n)} = \left|\frac{U_{out(n)}}{U_{in(n)}}\right| = \frac{1}{\left|1 + Z'_{(n)}Y''_{(n)}\right|}$$

Series and parallel resonant filters can be expressed as

$$Z'_{(n)} = jX'\left(n - \frac{1}{n}\right) \; Y''_{(n)} = jY''\left(n - \frac{1}{n}\right)$$

And the filtering is

$$f_{(n)} = \frac{1}{\left|1 - X'Y''\left(n - \frac{1}{n}\right)^2\right|} = \frac{n^2}{\left|n^2 - X'Y''\left(n^2 - 1\right)^2\right|}$$

The third harmonic in the input is one third and in the output it can be 2 %, thus

$$f_{(3)} = \left|\frac{U_{\text{out}(3)}}{U_{\text{in}(3)}}\right| = \frac{0{,}03U_1}{U_3} = \frac{0{,}03U_1}{U_1/3} = 0{,}06$$

$$f_{(3)} = \frac{1}{|1 - X'Y''(3 - 1/3)^2|} = 0{,}06$$

and we obtain

$$\frac{1}{1 - X'Y''(3 - 1/3)^2} = 0{,}06 \;\text{ or }\; \frac{1}{-1 + X'Y''(3 - 1/3)^2} = 0{,}06$$

which gives X'Y'' ≈ -2,203 or X'Y'' ≈ 2,203 and because it has to positive the latter one is selected.

$$Y'' = \frac{1}{L''\omega} = C''\omega = \frac{1{,}703}{X'} \approx 0{,}037$$

$$Y'' = \frac{1}{L''\omega} = C''\omega = \frac{2{,}203}{X'} \approx 0{,}0958$$

and we get C'' ≈304,9 μF and L'' ≈ 33,2 mH.

Question 5

Phase control or line-frequency modulation
-Devices used: diodes (no control needed) and thyristors (need turn on pulse)
-Modulation needs to be synchronized with supply frequency, diodes do this automatically, thyristors needs synchronization
-Delay causes phase shift between voltage and current in the supply side => reactive power needed from the supply system
-Switching frequency is low, there low order harmonics both on the output side (voltage) and input side, line current harmonics
-Because of low switching frequency practically no switching losses
-Example, e.g. Question 1

PWM
-Needs gate controlled devices like MOSFETs, IGBT which are turned on and off with separate control electronics => more complicated and expensive than phase control
-Relative on time of the conducting switches is adjusted, control voltage is compared to high frequency carrier, in three-phase converter space vector modulation can be used too
-High switching frequency increases the frequency of harmonics => filtering becomes easier, however switching losses are increasing too
-High switching frequency enables also fast reaction time on changes, e.g. output voltage can be adjusted fast
-All different conversions can be done by using PWM, more options than with phase control
-Example, Question 2 of 3.