Aalto University School of Electrical Engineering
Department of Communications and Networking
ELEC-E7450 Performance Analysis, Spring 2018

Examination
23.5.2018
Samuli Aalto

1. Consider an M/G/1-FIFO queue with arrival rate λ and IID service times that consist of a random number of IID phases,
$$S = \sum_{i=1}^{N} Y_i,$$
where phase Y_i obeys the exponential distribution with mean $E[Y_i] = 1$ and N is an independent random variable having a uniform distribution, $P\{N = 1\} = P\{N = 2\} = 1/2$.
   (a) For which values of λ the system is stable?
   (b) Give also the mean number of customers in the system when $\lambda = 1/2$.

2. Consider flow-level data traffic in a bottleneck link of an IP network. The traffic consists of elastic flows (such as file transfers using TCP), which arrive according to a Poisson process with rate $\lambda = 3$ flows/s. The flow sizes B_i are IID random variables with mean $E[B] = 30$ Mb, and the link capacity is C Mbps. Use the M/G/1-PS model to determine
   (a) the stability condition for the link capacity C;
   (b) the mean file transfer time for a randomly chosen flow when $C = 100$ Mbps.

3. Consider a Jackson network with $M = 2$ nodes. For each node $i = 1, 2$, new customers arrive at rate $\lambda/2$. The service times in both nodes are independent and exponentially distributed with mean $1/\mu$. Upon departure from node 1, the customer joins the queue of node 2 with probability 1/3 and leaves the whole network with probability 2/3. On the other hand, upon departure from node 2, the customer joins the queue of node 1 with probability 2/3 and leaves the whole network with probability 1/3. Assume further that $\lambda = 2/3$ and $\mu = 1$.
   (a) What is the average time that a customer spends in the system?
   (b) What is the average time that a customer who enters the network at node 1 spends in the system?

4. Consider a two-link network with capacities $C_1 = 200$ Mbps and $C_2 = 300$ Mbps. Route $k = 1$ uses both links, while route $k = 2$ uses only link 2. Let n_k denote the number of class-k flows. Determine the proportional fair (PF) inter-class allocations $\phi_k(n_1, n_2)$ for both classes k when $n_1 = 3$ and $n_2 = 2$.

5. Consider the following service system. New customers arrive according to a Poisson process with intensity λ. Customers are served one-by-one according to the FIFO service discipline. The service times are IID random variables obeying the exponential distribution with mean $1/\mu$. After each service the server needs a recovery interval, during which it is not able to serve any customers. The recovery intervals are IID random variables obeying the exponential distribution with mean $1/\delta$. The service of the customers continues as soon as the recovery interval is over. If there are no customers when a recovery interval ends, the server remains idle and ready to start service as soon as a new customer arrives. Let $X(t)$ denote the number of customers in the system at time t. In addition, let $Z(t)$ denote the state of the server at time t with $Z(t) \in \{0, 1, 2\}$ indicating that the server is idle (0), busy (1), or recovering (2). Draw the state transition diagram of the two-dimensional Markov process $(X(t), Z(t))$.