Aalto-yliopisto
Perustieteiden korkeakoulu

Matematiikan laitos Malmivuori

MS-A0202 Differentiali- ja integraalilaskenta 2

Loppukoe 19.6.2018 klo 16-19

Täytä selvästi jokaiseen vastauspaperiin kaikki otsaketiedot. Merkitse kurssikoodikohtaan opintojakson numero, nimi selvästi. Merkitse kaikki omat tietosi; nimi ja opiskelijanumero. Tutkinto-ohjelmakoodit ovat ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT.

Write on each paper clearly your name and your student number. Write also headings above; i.e. the name of the course, the course code and on which of the programs ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT are you studying; or if you have still another program which is not mentioned here, then write it.

Kokeessa saa käyttää korkeintaan funktiolaskinta. Näin ollen esim. graafiset ja ohjelmoitavat laskimet ovat kiellettyjä. Kokeessa ei saa käyttää muita apuvälineitä ongelmien ratkaisuun.

Koeaika on 3h.

You may use at most a functional Calculator. Thus e.g. grafical or a calculator that allows you the ability of programming is not allowed. No any other extra equipment is allowed for solivng the problems.

Exam time is 3 hours.

1. (6 p.) Onko funktio

$$f(x,y) = \begin{cases} \frac{5x^4y}{x^4+\exp(y)}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

jatkuva kaikissa pisteissä $(x,y) \in R^2$? Todista funktion jatkuvuus pisteessä $(0,0)$ määritelmän avulla, jos se on jatkuva.

Määrää osoittaisderivaatta $\frac{\partial f}{\partial y}(0,0)$ ja $\frac{\partial f}{\partial x}(0,0)$ määritelmän avulla, jos se on olemassa.

Is the function f which is defined by

$$f(x,y) = \begin{cases} \frac{5x^4y}{x^4+\exp(y)}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

continuous in every point $(x, y) \in R^2$? If function is continuous, prove it by using the definition of continuity at $(0, 0)$.

Calculate the partial derivative $\frac{\partial f}{\partial y}(0,0)$ and $\frac{\partial f}{\partial x}(0,0)$ by using the definition of the partial derivative if it exists.

2. (6 p.) Määrää Taylorin polynomi astetta 2 funktiolle $f(x, y) = e^{x^2+y}$, kun kehityspisteenä on $(1, 1)$.

Determine the Taylor polynomial of degree 2 for the function $f(x, y) = e^{x^2+y}$ near the point $(1, 1)$ (i.e. the polynomial has the same value as the function f at $(1, 1)$).

3. (6 p.) Määrää funktion $f(x, y) = 2\cos(x) + 3y^2$ lokaalit ääriarvot ja niiden laatu joukossa, jonka määrää ehto $x^2 + y^2 \le 1$.

Determine the local maximum and minimum points of the function $f(x, y) = 2\cos(x) + 3y^2$ in the set that defines the condition $x^2 + y^2 \le 1$.

4. (6p.) Laske integraali

$$\int\int_A f(x, y)dA,$$

missä $A \subset R^2$ on joukko, joka määräytyy ehdoista $x \ge \sqrt{2}$, $y \ge 0$, $y \le x$, $x^2 + y^2 \le 4$ ja funktio f on $f(x, y) = 4x$.

Evaluate the integral

$$\int\int_A f(x, y)dA,$$

where $A \subset R^2$ is the set, which is defined by the conditions $x \ge \sqrt{2}$, $y \ge 0$, $y \le x$, $x^2 + y^2 \le 4$ and the function f is $f(x, y) = 4x$.

Kaavoja ilman selityksiä / Equalities without explanations:

$$\bar{t}_1 = \mathbf{i} + \frac{\partial f}{\partial x}(a,b)\mathbf{k}, \quad \bar{t}_2 = \mathbf{j} + \frac{\partial f}{\partial y}(a,b)\mathbf{k}$$

$$\bar{n} = \bar{t}_2 \times \bar{t}_1 = \frac{\partial f}{\partial x}(a,b)\mathbf{i} + \frac{\partial f}{\partial y}(a,b)\mathbf{j} - \mathbf{k}$$

$$\lim_{(h,k)\to(0,0)} = \frac{f(a+h,b+k) - f(a,b) - hf_x(a,b) - hf_y(a,b)}{\sqrt{h^2+k^2}} = 0$$

$$D\,f(x) = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \frac{\partial y_1}{\partial x_3} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \frac{\partial y_2}{\partial x_3} \\ \frac{\partial y_3}{\partial x_1} & \frac{\partial y_3}{\partial x_2} & \frac{\partial y_3}{\partial x_3} \end{pmatrix}.$$

$$y'(x_0) = -\frac{\frac{\partial f}{\partial x}(x_0,y_0)}{\frac{\partial f}{\partial y}(x_0,y_0)}$$

$$f(a+h,b+k) = \sum_{m=0}^{\infty}\sum_{j=0}^{m} \frac{1}{j!(m-j)!} D_1^j D_2^{m-j} f(a,b) h^j k^{m-j}$$

$$Hess\, f(P) = \begin{pmatrix} f_{xx}(P) & f_{xy}(P) \\ f_{xy}(P) & f_{yy}(P) \end{pmatrix}$$

$$L(x_1,x_2,\ldots,x_n,\lambda_1,\lambda_2,\ldots,\lambda_m) = f(x_1,\ldots,x_n) - \lambda_1 g_1(x_1,\ldots,x_n) - \ldots - \lambda_m g_m(x_1,\ldots,x_n)$$

$$a = \frac{\overline{xy} - \overline{x}\,\overline{y}}{\overline{x^2} - (\overline{x})^2}, \quad \overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$b = \frac{\overline{x^2}\overline{y} - \overline{x}\,\overline{xy}}{\overline{x^2} - (\overline{x})^2}$$

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}(f(g(x)) = f'(g(x))g'(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = 2\cos^2(x) - 1$$
$$\sin^2(x) + \cos^2(x) = 1$$
$$\sin(\tfrac{\pi}{2} - x) = \cos(x)$$
$$\cos(\tfrac{\pi}{2} - x) = \sin(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\sin(0) = 0$$
$$\cos(0) = 1$$
$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right)$$
$$\sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} = \cos\left(\frac{5\pi}{4}\right)$$

$$e = 2,718281828459045$$

sin,cos summakaavat:

$$\begin{aligned}
\sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y) \\
\sin(x-y) &= \sin(x)\cos(y) - \cos(x)\sin(y) \\
\cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \\
\cos(x-y) &= \cos(x)\cos(y) + \sin(x)\sin(y)
\end{aligned}$$

Muita kaavoja

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{\frac{\partial(x,y)}{\partial(u,v)}}$$

$$dx\,dy = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

$$dx\,dy\,dz = \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| du\,dv\,dw$$

$$ds = |\mathbf{r}'(t)|dt$$

Apuyhtälöitä

$$ar^2 + br + c = 0 \qquad \text{tai} \qquad ar(r-1) + br + c = 0$$

molemmissa 2 juurta r_1, r_2, 3 tapausta:
(a) $r_1 \neq r_2 \in \mathbb{R}$, (b) $r_1 = r_2 \in \mathbb{R}$, (c) $r_{1,2} = k \pm i\omega \in \mathbb{C}$.

Numeerisia menetelmiä yhtälölle $x' = F(x,t)$:

- Euler: $x^{(k)} = x^{(k-1)} + hF(x^{(k-1)}, t^{(k-1)})$.
- Paranneltu Euler: $x^{(k)} = x^{(k-1)} + \frac{h}{2}\Big(c_1 + c_2\Big)$, missä

$$\begin{aligned}
c_1 &= F(x^{(k-1)}, t^{(k-1)}), \\
c_2 &= F(x^{(k-1)} + hc_1, t^{(k-1)} + h).
\end{aligned}$$

- Runge-Kutta: $x^{(k)} = x^{(k-1)} + \frac{h}{6}\Big(c_1 + 2c_2 + 2c_3 + c_4\Big)$, missä

$$\begin{aligned}
c_1 &= F(x^{(k-1)}, t^{(k-1)}), \\
c_2 &= F(x^{(k-1)} + \frac{1}{2}hc_1, t^{(k-1)} + \frac{1}{2}h), \\
c_3 &= F(x^{(k-1)} + \frac{1}{2}hc_2, t^{(k-1)} + \frac{1}{2}h), \\
c_4 &= F(x^{(k-1)} + hc_3, t^{(k-1)} + h).
\end{aligned}$$