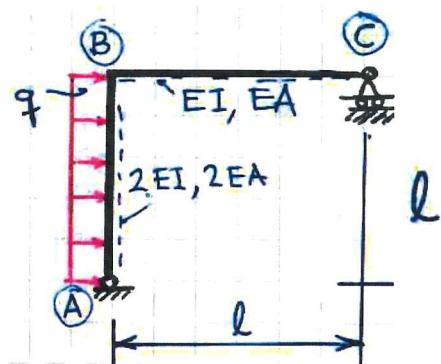


All the structures here are planar and linear elastic.

1. Statics [5 p]

a) Determine and draw accurately the diagrams of the distributions for the internal forces M and N . [3 p]

b) Determine the rotation at support C accounting both for bending and axial deformations. [2 p]



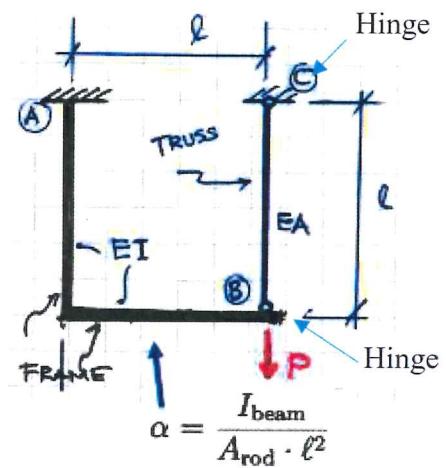
2. General Force Method [5 p]

AB is a frame working in bending only and BC is a rod with stretch or compression only.

Determine a) the axial force X_1 in the rod BC [4 points].

b) For the value of the parameter $\alpha = 4/3$, draw the distribution of the internal bending moment M and determine the axial force in the truss. [1 points].

Hint: 1) *Flexibility coefficients*: For the frame, consider only bending contributions and for the truss only the effects of axial deformations, respectively. 2) Express $X_1 = P \times \text{something}$ containing α .



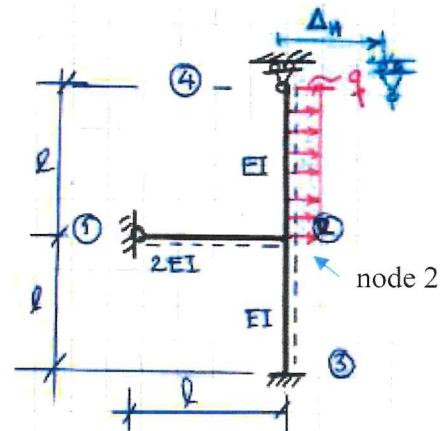
3. Slope-Deflection Method [a) & b): 5 pnts]

Determine

a) Rotation ϕ_2 at node 2 [4 pnts]

b) Bending moment at node 3. [1 pnt]

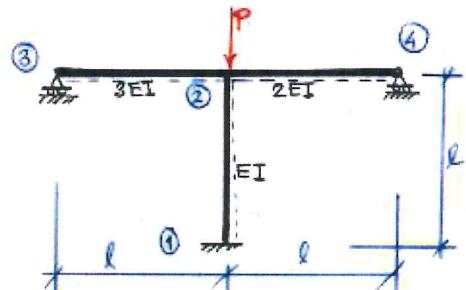
c) EXTRA [1 points] determine the horizontal displacement at node 4.



4. Buckling frames [5p]

Use Slope-Deflection Method and a) derive the explicit expression, in terms of Berry's stability functions, of the needed criticality condition for determining the critical buckling load [4 p].

b) Give a bracket (lower and upper bounds) for the value of critical buckling load using the Euler's basic cases [1p].



- N.B. Results presented without the logical steps needed to achieve them will be ignored even if they are correct.
- Number (numeroi juoksevasti) your answer papers 1(n) ... n(n) and, on each of them, write readably your name, family name and student number with the exam-date.

CIV-E1020 - Mechanics of Beam and Frame Structures Examination 23.10.2018

Duration: 3h

No formulary is allowed

Euler's basic buckling cases Eulerin perusnurjahdus				
$P_{cr} = \mu \frac{\pi^2 EI}{L^2}$				
Topaus	1	2	3	4
μ	0.25	1	2.046	4
	5			

$$M_{ij} = A_{ij}\phi_{ij} + B_{ij}\psi_{ij} - C_{ij}\psi_{ij} + \bar{M}_{ij} \quad M_{ij}^0 = A_{ij}^0\phi_{ij} - C_{ij}^0\psi_{ij} + \bar{M}_{ij}^0$$

Beam-column with constant flexural rigidity:

$$A_y = A_x = \frac{2\psi(kL)}{4\psi^2(kL) - \phi^2(kL)} \quad B_y = B_x = \frac{\phi(kL)}{4\psi^2(kL) - \phi^2(kL)} \frac{6EI}{L}$$

$$C_y = A_y + B_y \quad A_y^0 = C_y^0 = \frac{1}{\psi(kL)} \frac{3EI}{L}, \quad kL \equiv L \sqrt{\frac{P}{EI}}$$

Berry's functions:

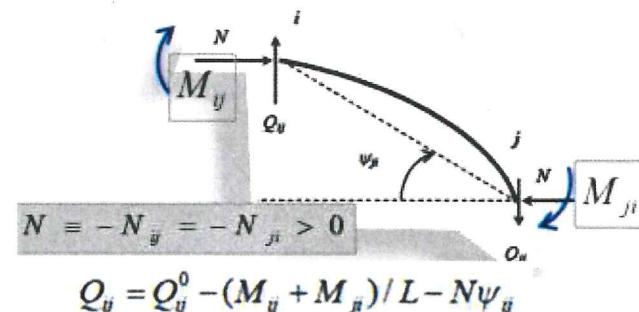
$$\text{Olkoon } \lambda \equiv kL, \quad \lambda \equiv kL$$

Puristettu sauva:

$$\phi(\lambda) = \frac{6}{\lambda} \left(\frac{1}{\sin \lambda} - \frac{1}{\lambda} \right), \quad \psi(\lambda) = \frac{3}{\lambda} \left(\frac{1}{\lambda} - \frac{1}{\tan \lambda} \right), \quad \text{ja} \quad \chi(\lambda) = \frac{24}{\lambda^3} \left(\tan \frac{\lambda}{2} - \frac{\lambda}{2} \right)$$

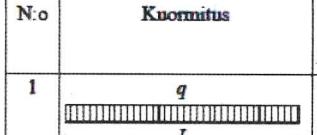
Vedetty sauva:

$$\phi(\lambda) = \frac{6}{\lambda} \left(-\frac{1}{\sinh \lambda} + \frac{1}{\lambda} \right), \quad \psi(\lambda) = \frac{3}{\lambda} \left(-\frac{1}{\lambda} + \frac{1}{\tanh \lambda} \right), \quad \text{ja} \quad \chi(\lambda) = \frac{24}{\lambda^3} \left(-\tanh \frac{\lambda}{2} + \frac{\lambda}{2} \right)$$



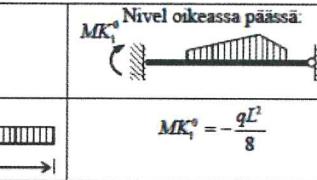
$$\bar{M}_{12} \equiv MK_1$$

$$\bar{M}_{ij}, \bar{M}_{ji}$$

N:o	Kuormitus	Kiinnitysmomentit:
1		$MK_1 = -\frac{qL^2}{12}, MK_2 = \frac{qL^2}{12}$
2		

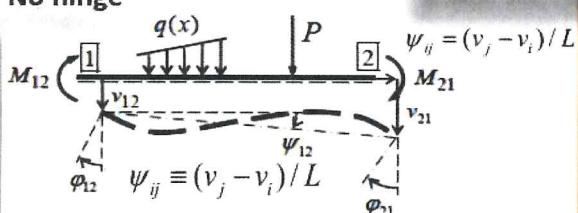
$$\bar{M}_{ij}$$

$$\bar{M}_{ji}$$

N:o	Kuormitus	Nivel oikeassa päässä: MK_1^0	Nivel vasemmassa päässä: MK_2^0
1		$MK_1^0 = -\frac{qL^2}{8}$	$MK_2^0 = \frac{qL^2}{8}$

The stiffness equation relating the end-moments to the end-displacements

No hinge



$$M_{ij} = a_{ij}\phi_{ij} + b_{ij}\phi_{ji} - c_{ij}\psi_{ij} + \bar{M}_{ij}, \quad i \neq j$$

$$a_{ij} = \frac{4EI}{L}, \quad b_{ij} = \frac{2EI}{L}, \quad c_{ij} = \frac{6EI}{L} \quad (EI\text{-constant})$$



Fixed end-moment resulting from external mechanical loading, look from tables

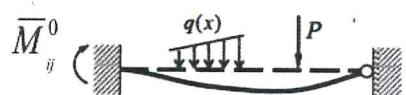
If you are using lecture's notations

One node is hinged

The 0 is a superscript "0" means that the support at end j is hinged

$$M_{ij}^0 = a_{ij}^0\phi_{ij} - c_{ij}^0\psi_{ij} + \bar{M}_{ij}^0$$

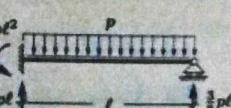
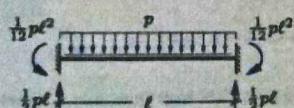
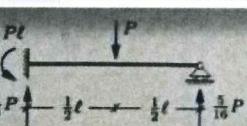
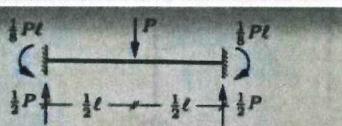
$$a_{12}^0 = c_{12}^0 = \frac{3EI}{L} \quad \psi_{ij} = (v_j - v_i)/L$$



Fixed end-moment resulting from external mechanical loading, look from tables

If you are using Krenk's textbook notations

Bending Moments (pay attention to the sign convention to convert to Fixed-End-Moments)



If you are using Krenk's textbook notations

