# ELEC-E8116 Model-based control systems

## Intermediate exam 1. 25. 10. 2018

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are three (3) problems and each one must be answered.*
- *No literature is allowed. A function calculator can be used.*

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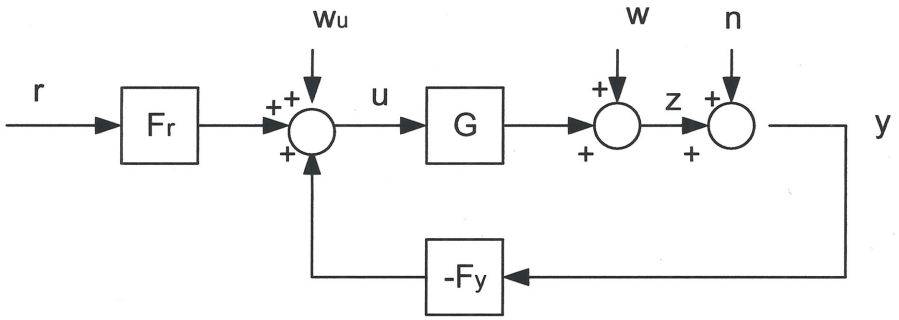
1. Explain briefly the following concepts

   - Singular value decomposition and singular values
   - Input and output directions of a multivariable system
   - Minimal realization of a multivariable transfer function
   - Multiplicative uncertainty model
   - Robust stability

**Note for problems 2 and 3**: When writing equations with matrices remember that a square matrix can have inverse $A^{-1}$ (if exists), but you cannot divide by a matrix ($1/A$ is illegal operation for a matrix). Also note that for matrices $AB \neq BA$ except for some rare special cases.

2. Consider a **multivariable** control configuration, where signal $y$ is $m$-dimensional and signal $u$ $n$-dimensional ($m$ and $n$ are positive integers).




   a. What are the dimensions of signals $r$, $w_u$, $w$, $n$ and matrices $G$, $F_y$, $F_r$?
   b. Write equations to show how $z$ depends on the input variables $r$, $w_u$, $w$, $n$. Identify the *loop transfer function*, *closed loop transfer function*, *sensitivity function* and *complementary sensitivity function*.
   c. Define $e = r\text{-}z$ and write equations showing how $e$ depends on the input variables. Use the specific transfer functions of part b when appropriate. Explain, how *loop shaping* can be linked to the result and what it means.

3. **a**. Let $A$ and $B$ me matrices of dimensions $m$ x $n$ and $n$ x $m$ respectively ($m$ and $n$ are positive integers). Prove the identity

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$$(I + AB)^{-1} A = A(I + BA)^{-1}$$

where the inverse matrices are assumed to exist and $I$ :s are identity matrices of appropriate dimensions. What is this identity called?

**b**. $L$, $S$ and $T$ are standard symbols used in the couse. Prove that in all frequencies it holds

- $S + T = \mathrm{I}$
- $LS = SL$
- $(I + L)^{-1} L = I - (I + L)^{-1}$