Aalto University School of Electrical Engineering
Department of Communications and Networking
ELEC-C7210 Modeling and analysis of communication networks, Autumn 2018

Examination
24.10.2018
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Please answer to all five (5) questions. Use of calculators/additional material is not allowed in the exam.

1. Assume that X is exponentially distributed with parameter λ. One of the claims below is the memoryless property of the exponential distribution and other claims are erroneous:

$$P\{X \geq i+j\} + P\{X \geq i\} = P\{X \geq j\}, \text{ for all } i,j \geq 0, \quad \text{(a)}$$
$$P\{X \geq i+j \mid X \geq i\} = P\{X \geq j\}, \text{ for all } i,j \geq 0, \quad \text{(b)}$$
$$P\{X \geq i+j \text{ and } X \geq i\} = P\{X \geq j\}, \text{ for all } i,j \geq 0. \quad \text{(c)}$$

   (a) Which of the claims, a), b), or c) is the memoryless property?

   (b) Derive the memoryless property for the exponential distribution, for which $P\{X \geq k\} = e^{-\lambda k}$.

   (c) Let us assume that X models the length (=holding time) of a telephone call, and $\lambda = 1/2$ (1/min). Assume that the call has lasted already 5 minutes and let X^R denote the random variable for the remaining call holding time. What is the expectation (or mean value) of X^R, i.e. $E[X^R]$?

2. Consider a system with 3 parallel servers and 4 waiting places. The average service time is 1 minute. Customers are served in their arrival order. Assume it has been measured that on average 2 customers per minute leave from the system after having completed their service and that on average there are 6 customers in the system (totally in service or waiting). What is the average waiting time in the system?

3. Consider the M/M/2/4 model with mean customer interarrival time of $1/\lambda$ time units and mean service time of $1/\mu$ time units. Let $X(t)$ denote the number of customers in the system at time t, which is a Markov process.

   (a) Draw the state transition diagram of $X(t)$.

   (b) Derive the equilibrium distribution of $X(t)$. When is the system stable, i.e., is there any condition for existence of a unique equilibrium distribution in this case?

   (c) Assume that $\lambda = \mu$. What is the probability that an arriving customer must wait before service?

4. Consider elastic data traffic carried by a 100-Mbps link in a packet switched network. Use a pure sharing system model with a single server. New flows arrive according to a Poisson process at rate 70 flows per second, and the sizes of files to be transferred are independently and exponentially distributed with mean 1 Mbit. Let $X(t)$ denote the number of ongoing flows at time t.

   (a) What is the traffic load?

   (b) Derive the equilibrium distribution of $X(t)$. Is there any condition for stability, i.e., is there any condition for existence of a unique equilibrium distribution in this case?

   (c) What is the throughput of a flow?

Last question on the other side of the paper

5. Consider a system of two components in series where the failure of either of the components results in the other component taken offline immediately as well. That is, if component 1 fails, component 2 is immediately put offline, thus it cannot fail during the repair of component 1 and vice-versa. Failure rate of component 1 is λ_1, repair rate μ_1 and, correspondingly, for component 2, λ_2 and μ_2.

   a) Model the behavior of the system as a Markov process and draw the state transition diagram.

   b) Calculate the equilibrium distribution.

   c) What is the average availability of the system?

   d) What is the MTTF of the system?