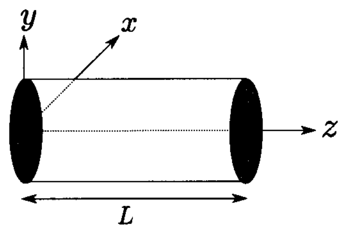
Aalto University
School of Science

PHYS-E0413 Theoretical Mechanics, Autumn 2017, Midterm exam 2.

1. Explain shortly the concepts:
   (a) Dynamic viscosity
   (b) Bulk viscosity
   (c) Laminar flow
   (d) Young's modulus
   (e) Elastic wave
   (f) Yield stress
2. Explain the main concepts of modeling a flowing fluid.
3. A fluid flows in a long pipe with a circular cross section of radius a. Assume that the flow is stationary and incompressible and that there is no volumetric forces present. The pressure at $z = 0$ is p_1 and at $z = L$ the pressure is p_2. The density of the fluid is ρ and the kinematic viscosity ν.



   (a) Solve for the pressure field p and the velocity field v of the fluid.
   (b) Calculate the drag force the fluid exerts on the wall of the cylinder for $0 \leq z \leq L$.
   (c) Calculate the mass flow rate through the pipe.
4. Give an example of a strain tensor for which there is
   (a) an increase in volume.
   (b) extension in the z-direction, but overall decrease in volume.
   (c) shear strain, but no volume change.

   For the strain tensor of (a) write down the corresponding stress tensor for an isotropic solid given the Lamé coefficients μ and λ. [1]
5. A two-dimensional problem of a rectangular bar stretched by uniform end loadings results in the following constant strain field:

$$\epsilon = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & -C_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

   where C_1 and C_2 are constants. Assuming the field depends only on x and y, integrate the strain-displacement relations to determine the displacement components and identify any translation and rotation terms.

Equations

- Euler's equation:

$$\frac{\partial}{\partial t}\vec{v} + (\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{\rho}\nabla p + \vec{f}$$

- Mass continuity:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

- Navier-Stokes equation:

$$\frac{\partial}{\partial t}\vec{v} + (\vec{v}\cdot\nabla)\vec{v} = \vec{f} - \frac{1}{\rho}\nabla p + \frac{\eta}{\rho}\nabla^2\vec{v} + \frac{1}{\rho}\left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla\cdot\vec{v})$$

- Vector identities:

$$(\vec{v}\cdot\nabla)\vec{v} \equiv (\nabla\cdot\vec{v})\vec{v} - \vec{v}(\nabla\cdot\vec{v})$$

$$(\vec{v}\cdot\nabla)\vec{v} \equiv \nabla(\frac{1}{2}v^2) - [\vec{v}\times(\nabla\times\vec{v})]$$

- Strain:

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

- Strain for a Hookean material

$$\epsilon_{ij} = -\frac{1}{9K}\delta_{ij}\,\mathrm{Tr}(T) - \frac{1}{2\mu}\left(T_{ij} - \frac{1}{3}\delta_{ij}\,\mathrm{Tr}(T)\right)$$

- Stress for a Hookean material

$$T_{ij} = -K\delta_{ij}\,\mathrm{Tr}(\epsilon) - 2\mu\left(\epsilon_{ij} - \frac{1}{3}\delta_{ij}\,\mathrm{Tr}(\epsilon)\right)$$

$$T_{ij} = -\lambda\delta_{ij}(\nabla\cdot\vec{u}) - \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$