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Department of Computer Science
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CS-C2150 Theoretical Computer Science — Spring 2018

Exam April 4th, 2018, 17:00–20:00

Note: Use of calculators, tablets, laptops, etc, is not allowed in the exam. Use of textbooks, lecture notes, or personal notes is not allowed either.
Note: If you have not completed both your computerized home assignments and your Coq mandatory assignment, you will not pass the course.
Total points: $6 + 16 + 15 + 20 + 12 + 1 = 70$.

1. (6 points)

   Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$.

   (a) Is A a subset of B?

   (b) Is B a subset of A?

   (c) What is $A \cup B$?

   (d) What is $A \cap B$?

   (e) What is $A \times B$?

   (f) What is the powerset of B?

2. (16 points)

   In all answers below, make sure that your automata are *complete*, i.e., that they have transitions defined from every state and for every input symbol.

   (a) Design a deterministic finite automaton that recognizes the language

   $$\{w \in \{a, b\}^* \mid w \text{ starts with the substring } aab\}.$$

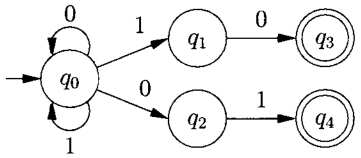
   You can assume that the alphabet is $\{a, b\}$.

   (b) Design a deterministic finite automaton that recognizes the language

   $$\{w \in \{a, b\}^* \mid w \text{ does not contain the substring } aab\}.$$

   You can assume that the alphabet is $\{a, b\}$.

   (c) Let us consider the following non-deterministic automaton over the alphabet $\{0, 1\}$:



   Describe the laguage recognized by the automaton verbally, in one or two sentences. Give a deterministic finite automaton that recognizes the same language.

3. (15 points)

(a) Give a regular expression that describes the language

$$L = \{w \in \{a,b\}^* \mid w \text{ begins and ends with the same symbol}\}$$

Note: the empty string does *not* belong to the above language.

(b) Consider the regular expression $(b \cup (aa^*b))^*$ over the alphabet $\{a,b\}$. Give the complete deterministic finite state automaton with *minimal number of states* that recognizes the language described by this regular expression.

(c) Give a regular expression that describes the language

$$L = \{w \in \{a,b\}^* \mid w \text{ does not contain the substring } aa\}$$

4. (20 points)

Let $G = (V, \Sigma, R, S)$ be a context-free grammar, where $V = \{S, T, U\}$, $\Sigma = \{0, \#\}$, and R is the set of rules:

$$\begin{aligned} S &\rightarrow TT \mid U \\ T &\rightarrow 0T \mid T0 \mid \# \\ U &\rightarrow 0U00 \mid \# \end{aligned}$$

(a) Can the string # be generated by the grammar? If so, give a parse tree for it. If not, provide explanation.

(b) Can the string 00# be generated by the grammar? If so, give a parse tree for it. If not, provide explanation.

(c) Can the string 0##0 be generated by the grammar? If so, give a parse tree for it. If not, provide explanation.

(d) Give two more strings, different from all strings above and from each other, that can be generated by the grammar: show their derivations.

(e) Give two more strings, different from all strings above and from each other, that cannot be generated by the grammar. The strings must use the alphabet Σ above. Provide explanations why they cannot be generated by G.

(f) Prove that the language generated by G is not regular.

(g) Give a PDA (pushdown automaton) that recognizes the same language as that generated by G.

5. (12 points)

(a) Define the concepts "Turing-recognizable language" (also called "recursively enumerable language") and "Turing-decidable language" (also called "recursive language").

(b) Consider the language

$$\{x \in \{0,1,2,3,4,5,6,7,8,9\}^* \mid x \text{ is a product of two prime numbers}\}.$$

For instance, the string 15 is in the language as $3 \times 5 = 15$ but 16 is not in the language. Is the language Turing-decidable? Justify your answer.

(c) Prove the following claim either correct or incorrect: Let L_1 and L_2 be languages over an alphabet Σ. If L_1 is context-free and L_2 is Turing-decidable, then also the language $L_1 \cap L_2$ is context-free.

6. (1 point)
At what time did you finish answering the exam questions?