The five problems in this examination are each graded on a scale from zero to six points. Please justify your answers and write your name, student number and degree program clearly on each answer sheet.

1. Explain briefly the following concepts:

a)	Futures contract.	(1p)
b)	The expectation hypothesis of interest rates.	(1p)
c)	Risk neutral probability.	(1p)
d)	Short selling of financial assets.	(1p)
e)	Minimum variance hedge.	(1p)
f)	Sharpe ratio.	(1p)

2. Are the following statements true or false? Justify your answer.

   a) If the maturity of Bond A is longer than that of Bond B, then the duration of Bond A will be higher than that of Bond B. (1 p)

   b) The price of an American put option can be lower than that of a similar European put option (similar = same underlying asset, strike price, expiry date). (1p)

   c) In the standard capital budgeting problem, choosing projects in the order of decreasing benefit-cost ratios (i.e., project with the highest ratio is selected first) always leads to the selection of the project portfolio which offers the highest possible total benefit for the available budget. (1p)

   d) The delta (Δ) of a European call option can assume values outside of the interval [0,1]. (1p)

   e) The premium of a European put option increases if the price of the underlying asset becomes more volatile. (1p)

   f) If the expectations hypothesis of interest rates holds, the prices of futures and forward contracts are equal for the delivery of the same commodity at the same time and place. (1p)

3. Assume that a company is liable to pay 2 million euros after a year and 3 million euros after 4 years and that the spot rates for the next five years are as follows:

Year	1	2	3	4	5
Liability	2	-	-	3	-
Spot rate	3%	4%	5%	6%	7%

   (a) Determine the present value and duration of the liabilities of the company. (2p)

   (b) The company seeks protection from possible parallel shifts in spot rates by investing in an immunizing portfolio consisting of bonds A and B. Bond A has a coupon rate of 6% and matures in 2 years (annual coupon payments). Bond B is a zero-coupon bond with maturity in 4 years. How much must the company invest in bonds A and B, respectively? (4p)

4. Suppose that the only stocks in the market are A and B whose parameters are below. The correlation between the returns of the two stocks is $\rho_{AB} = 0.2$.

Stock	Number of stocks in the market	Price (€)	Expected return	Standard deviation of return
A	500	5.00	11%	25%
B	250	10.00	8%	15%

The investors have access to unlimited borrowing and lending at the risk-free rate r_f. The market is assumed to satisfy the assumptions of CAPM (Capital Asset Pricing Model).

(a) What are the expected return and standard deviation of the market portfolio? (2p)

(b) What are the β's of stocks A and B? (2p)

(c) What is the risk-free rate? (2p)

5. (a) Consider a finite annuity with n periodic payments A and present value P. Assume that the interest rate r in each period is the same. Show that

$$A = \frac{r(1+r)^n P}{(1+r)^n - 1},$$

where n is the number of payment periods. (2p)

(b) Consider n assets with random returns r_i with expectations $\bar{r}_i$ and covariances $\sigma_{ij}, i, j = 1, 2, \ldots, n$ (and variances $\sigma_i^2 = \sigma_{ii}$). Show that all efficient portfolios are characterized by portfolio weights $w_i, i = 1, \ldots, n$ which satisfy the equations

$$\sum_{j=1}^{n} \sigma_{ij} w_j - \lambda \bar{r}_i - \mu = 0, \quad \forall i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} w_i \bar{r}_i = \bar{r}$$

$$\sum_{i=1}^{n} w_i = 1,$$

where $\bar{r}$ is the required rate of portfolio return. (2p)

(c) The solution to the Black-Scholes equation for a European call option with strike price K and expiry date T is

$$C(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2), \text{ where}$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S/K) - (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t},$$

where S is the price of the underlying asset, σ is the price volatility of this asset, r is the risk free interest rate and $N(x)$ is the standard cumulative normal probability distribution

$$N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy.$$

State and interpret the values of $C(S,t)$ when (i) $t = T$, and when (ii) $T = \infty$. (2p)