Aalto University
School of Science

PHYS-E0419 - Dynamics of particles, fluids and solids
Midterm exam Friday 26.10.2018

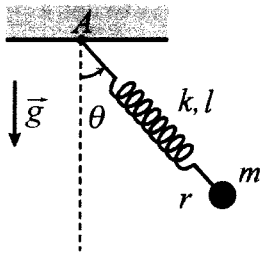
1. Explain following concepts biefly (max 1p each)
   (a) Holonomic constraint
   (b) Lagrange equation
   (c) Hamiltonian and its relation to Lagrangian
   (d) Cyclic coordinate
   (e) What does it mean if we say the lagrangian has a symmetry?
   (f) Phase space

2. A particle of mass m moves vertically in a gravitational potential $V(y) = mgy$ ($g > 0$). It starts (at $t = 0$) from $y = 0$ and at $t = T$ is at $y(T) = 0$. Let us assume a trial path

$$y(t) = at^2 + bt + c, \tag{1}$$

   where a, b, and c are taken as unknown parameters.
   a) Calculate the action from $t = 0$ to $t = T$. (3p)
   b) Using Hamilton's principle determine what a, b, and c should be. (3p)

3. A particle of mass m is suspended from the point of support A with a massless spring (spring constant k so spring force is proportional to k, natural length l) illustrated in the figure beside. The particle can move on the plane of the figure. The gravitational field is constant $\vec{g}$.



   where r is the length of the spring and θ is the angle between the linear spring and the vertical.
   a) Find the Lagrangian of the system using θ and r as generalized coordinates. (2p)

   b) Find Lagrange's equations of motion. (2p)

   c) Solve for the normal mode frequencies assuming small oscillations around equilibrium. (2p)

4. For notational simplicity assume we have only one generalized coordinate q and generalized momentum p. Poisson bracket of two functions of (q, p) is defined as

$$\{f_1, f_2\} \equiv \frac{\partial f_1}{\partial q}\frac{\partial f_2}{\partial p} - \frac{\partial f_1}{\partial p}\frac{\partial f_2}{\partial q}. \tag{2}$$

a) Show that the time-evolution of function $f(q, p, t)$ is given by $df/dt = \{f, H\} + \partial f/\partial t$, where H is the Hamiltonian. (You can assume Hamilton's equations known.) (3p.)
b) Make a transformation between coordinates and momenta $(q, p) \rightarrow (Q, P)$ (assume again only one pair for simplicity). If $Q = q^n$ and $P = (p/2q)^m$, how could you choose n and m for the transformation to be canonical? (3p.)

Remember to answer in english unless you have a special permission to use some other language. Write your name, student number, study program, course code, and the date in all your papers. Use of calculators is forbidden.