TENTTI / TENT / EXAM

Kurssikoodi / Kurskod / Course code	MS-E2148	Kurssin nimi / Kursnamn / Course name	Dynamic Optimization
Tentin päivämäärä / Tent datum / Exam date	12.2.2018	Tentin kesto / Provtid / Exam duration	3h
Vastuuopettaja / Ansvarig lärare / Responsible teacher	Kimmo Berg	Tenttipaperin kieli(kielet) / Tentpapperets språk / Language(s) of the exam paper	ENGLISH
☐ Välikoe / Mellanprov / Midterm exam		☒ Tentti / Tent / Final exam	

SALLITUT APUVÄLINEET	TILLÅTNA HJÄLPMEDEL	ALLOWED MATERIAL
Tentissä saa käyttää vain kynää, kumia, viivainta ja harppia sekä niitä apuvälineitä jotka ovat merkitty sallituiksi oheiseen listaan.	I tenten får man använda enbart penna, radergummi, linjal och passare samt hjälpmedel som är markerat som tillåtet i listan nedan.	In the exam it is only allowed to use a pen, eraser, ruler and compass and the material that is marked as allowed in the list below.

	Sallittu Tillåten Allowed	Kielletty Förbjuden Forbidden
Funktiolaskin / Funktionsräknare / Regular calculator	☒	☐
Graafinen laskin (tyhjennettävä) / Grafräknare (bör tömmas) / Graphical calculator (to be emptied)	☒	☐
Laininen: kaava- ja taulukkokokoelma (valvojat jakavat) / formel och tabellsamling / formulary and table compendium	☐	☒
Mellin: kaava- ja taulukkokokoelma (opiskelijat tuovat itse) / formel och tabellsamling / formulary and table compendium	☐	☒
Muu materiaali (materiaali eriteltynä ja ohjeet sen tarkastukseen alla) / Övrig material (nedan) / Other material (below)	☐	☒

MUUT OHJEET / ÖVRIGA ANVISNINGAR / OTHER INSTRUCTIONS

☒ Tenttipaperi on palautettava valvojille Tentpapperet bör inlämnas åt övervakarna Exam paper must be returned	☐ Tenttipaperin saa viedä mukanaan Tentpapperet får tas med The exam paper can be taken

Kirjoita selvällä käsialalla jokaiseen vastauslomakkeeseesi tentin päivämäärä ja sali, kurssikoodi ja -nimi, opiskelijanumerosi, nimesi, koulutusohjelmasi ja allekirjoituksesi.

Jätä apuvälineet joita ei tarvita tentissä (kuten kännykät tai muut elektroniset laitteet) salin sivuille tai valvojille.

Varmista että nimesi kirjataan osallistujalistaan palauttaessasi vastauksesi. Tämä todistaa palautuksesi mikäli vastauslomakkeesi katoavat.

Suttupaperit merkitään kirjoittamalla paperiin suurilla kirjaimilla sana "SUTTU" ja vetämällä henkselit kirjoitusta sisältävien sivujen yli. Suttupapereita ei arvostella.

Tentissä noudatetaan yliopiston tenttiohjesääntöä.

Tenttikysymykset alkavat seuraavalta sivulta. KÄÄNNÄ SIVUA VAIN LUVALLA!

Skriv tydligt i varje svarsblankett tentens datum och sal, kurskod och -namn, samt ditt studienummer, namn, utbildningsprogram och underskrift.

Lämna de hjälpmedel som inte behövs i tenten (bl.a. mobiltelefoner och övriga elektroniska apparater) vid salens sidor eller åt övervakarna.

Försäkra att ditt namn skrivs på deltagarlistan när du inlämnar din prestation. Det bevisar din inlämning ifall dina svarsblanketter försvinner.

Klotterpapperen markeras med att skriva med stora bokstäver ordet "SUTTU" och med att dra ett kryss över varje sida som innehåller skrift. Klotterpapperen bedöms ej.

I tenten följer man universitetets tentamensregler.

Tentfrågorna börjar på nästa sida. VÄND SIDA ENBART MED TILLSTÅND!

Write clearly on every answer sheet the exam date and hall, course code and name, and your study number, name, study programme and signature.

Leave material that is not needed in the exam (e.g. mobile phones or other electronical devices) on the sides of the hall or to the supervisors.

Ensure that your name is written in the list of participants when returning your answers. It proves the returning if your answer sheets get lost.

Scratch papers are marked with the word "SUTTU" in capital letters and by drawing a large cross over all pages that have writing on them. Scratch papers are not graded.

The university's exam regulations are followed in the exam.

Exam questions start on the next page. TURN PAGE ONLY WITH PERMISSION!

Exam 12.2.2018

1. Explain shortly the following concepts:
   a. cost-to-go function
   b. transversality conditions
   c. open-loop control
   d. singular interval
   e. costate variable
   f. stopping problem.

2. a) Find the extremal of the functional

$$J(x) = \int_0^1 [\dot{x}(t)^2/2 + x(t)(\dot{x}(t) + 1)]dt,$$

using calculus of variations and explain what you are doing. $x(0) = 0$ and $x(1)$ is free.

   b) Find the extremal of the functional

$$\int_{-1}^1 [x(t)^2(1 - \dot{x}(t))^2]dt,$$

and explain what you are doing. $x(-1) = 0$ and $x(1) = 1$.

3. A student wants to pass an exam that is held after T time steps. Let us assume that the level of knowledge $k(t)$ increases with a speed that is proportional to the study effort $w(t)$. On the other hand, the rate of forgetting is directly proportional to the level of knowledge. The student attempts to minimize the time spent to study. This situation can be explained with the optimization model

$$\min\ J = \int_0^T w(t)dt$$

$$\dot{k}(t) = b\sqrt{w(t)} - ck(t),$$

where b and c are positive coefficients of proportionality, and $w(t) \geq 0, k(0) = k_0, k(T) = k_T > k_0$. Find the optimal study strategy $w(t)$.

PLEASE TURN!

$2x - 4x\dot{x} + 2\dot{x}^2 x = -4\dot{x} + 4\dot{x}^2 + 2x^2\ddot{x}$

4. a) Explain the relationship between δx_f, $\delta x(t_f)$ and δt_f and draw a figure.

   b) Italian brothers Mario and Luigi are hired to maintain the pipes of the building. The pipes can be working, leaking or broken. The men can either maintain the pipes, do nothing or fix them. Working pipes start leaking with probability 20% and get broken by 10%. Maintaining the pipes (working or leaking) cost 10 but halve the probabilities of failure. Leaking pipes double the probability of break down. Broken pipes must be fixed with cost 50. Leaking pipes cost 5. Fixing leaking pipes cost 20. All costs are per time unit. Model and solve the infinite horizon problem with discount factor $\alpha = 0.9$ using policy iteration with initial policy where the pipes are maintained. How much better is the optimal policy compared to the initial policy? Hint: the initial policy should give value $(I - \alpha P)^{-1} g \approx (137.6, 161.3, 173.9)$.

5. Model and solve a simple version of American football problem with dynamic programming. Show few steps of calculations how you would solve the problem. The problem is to find the best strategy to gain 10 yards in 3 downs (tries). The coach has two strategies: running play (ball is carried by running) or passing play (ball is thrown). Running plays have higher probability of success but offer smaller yards gained. Passing plays offer more yardage but the passes may be caught by opponent and then the ball is lost. For example, a successful outcome could be a gain of 4 yards by running, a loss of 2 yards by running and then gain os 12 yards by passing (4-2+12>10). A failing outcome could be a loss of ball by passing or gain of 2+4+3 yards by running (2+4+3<10).

HJB: $0 = J_t + \min_{u(t)} \{g + J_x^T f\}$
E-L: $0 = g_x - \frac{d}{dt}(g_{\dot{x}})$
Hamiltonian: $H = g + p^T(t) f(x(t), u(t), t)$
costate: $\dot{p}(t) = -\frac{\partial H}{\partial x}$
free state: $0 = g_{\dot{x}}$ or $h_x - p = 0$
free time: $0 = g - g_{\dot{x}}\dot{x}$ or $H + h_t = 0$
goal: $0 = g + \left[\frac{\partial g}{\partial \dot{x}}\right]^T \left[\frac{d\theta}{dt} - \dot{x}\right]$ or $H + h_t + (h_x - p)^T \frac{d\theta}{dt} = 0$
W-E: $g_{\dot{x}}$ and $g - g_{\dot{x}}\dot{x}$ continuous