

ELEC-E8116 Model-based control systems

Full exam 11. 12. 2018

- Write the name of the course, your name, your study program, and student number to each answer sheet.
- There are five (5) problems and each one must be answered.
- No literature is allowed. A function calculator can be used.
- Your solutions must contain enough information to see how you have solved the problems.

1. Explain briefly the following concepts

- Principle of optimality (in dynamic programming)
- Singular value decomposition
- Waterbed effect
- Robust stability
- Small gain theorem

2.a. Draw a schema of the "two-degrees-of-freedom" control configuration.

Define the concepts *sensitivity function* and *complementary sensitivity function* for it.

2.b. Consider a SISO-case. Determine the region in the complex plane where $|S|$ is smaller than, equal to, or larger than 1. How can the result be explained in view of control performance?

3. Consider a MIMO system with the transfer function matrix

$$G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{3}{s+2} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

- Determine the poles and zeros of the above system.
What conclusions can be made with respect to control?
- Calculate the *Relative Gain Array* RGA at zero frequency in the above example case.
What conclusions can be made with respect to control?

4. Consider a nominal SISO plant $G(s)$ with an *additive* uncertainty Δ

$$G_0(s) = G(s) + \Delta(s), \quad \|\Delta(s)\|_{\infty} \leq 1$$

Note that $G_0(s)$ is the true model and that the uncertainty is not multiplicative (like the one discussed in the lectures). The system is controlled by a 2 DOF controller with $F_r = 0$ and F_y . Derive a condition for the closed-loop system to be robustly stable.

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5. Consider the 1.order process $G(s) = \frac{1}{s-a}$, which has a realization

$$\begin{aligned}\dot{x}(t) &= ax(t) + u(t), \quad x(0) = x_0, \quad a \text{ constant} \\ y(t) &= x(t)\end{aligned}$$

so that the state is directly measurable . It is desired to find a control that minimizes the criterion

$$J = \frac{1}{2} \int_0^{\infty} (x^2 + Ru^2) dt \quad (R > 0)$$

Determine the optimal control law and the closed loop state equation. Is the closed loop system stable, when the process is i. stable, ii. unstable?

Some formulas that might be useful:

$$\dot{x} = Ax + Bu, \quad t \geq t_0$$

$$J(t_0) = \frac{1}{2} x^T(t_f) S(t_f) x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} (x^T Q x + u^T R u) dt$$

$$S(t_f) \geq 0, \quad Q \geq 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q, \quad t \leq t_f, \quad \text{boundary condition } S(t_f)$$

$$K = R^{-1}B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2} x^T(t_0) S(t_0) x(t_0)$$