CHEM-E7155 Production Planning and Control
Exam, 10.12.2018

1. **Answer shortly the following questions: (3p. – 1p. per question)**

A) What are the general elements of the optimization algorithms for linear and nonlinear programming to formulate a problem?

B) Give definition of the Convex Programming. Provide an example of optimization problem, which is not a convex programming.

C) What is the difference between the linear programming and integer programming?

2. **Linear and nonlinear programming: (4p.)**

A) Linear programming (3p.)

Solve the following problem using the simplex method:

$$\max 2x_1 - x_2 + x_3$$
$$x_1 - x_2 + 3x_3 \leq 4$$
$$2x_1 + x_2 \leq 10$$
$$x_1 - x_2 - x_3 \leq 7$$
$$x_1, x_2, x_3 \geq 0$$

B) Nonlinear programming (1p.)

Consider the following two-variable unconstrained optimization problem:

$$\text{Maximize } f(x) = 2x_1x_2 + x_2 - x_1^2 - 2x_2^2$$

Starting from the initial trial solution $x^{(0)} = (x_1^{(0)}, x_2^{(0)}) = (1,1)$, apply a single iteration of the gradient search procedure to obtain an approximation solution.

3. **Dynamic Programming (3p.)**

A manager of a chemical manufacturing company has decided to distribute six (6) chemical engineers to three (3) different production departments. He has planned to assign at least one (1) chemical engineer to each department. It is assumed that each chemical engineer can work only in one (1) department. The following table shows the estimated increase in profit of each department according to the number of the assigned chemical engineers. Determine the number of chemical engineers that must be allocated to each department to maximize the total profit.

Number of Chemical Engineer	Production Department		
	1	2	3
1	4	3	5
2	6	6	7
3	9	8	10
4	11	10	12

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4. Transportation simplex: (3p.)

Consider the transportation problem having the following parameter table:

		Destination				
		1	2	3	4	Supply
Source	1	8	6	10	9	35
	2	9	12	13	7	50
	3	14	9	16	5	40
Demand		45	20	30	30	

Starting with the northwest corner rule, apply the transportation simplex method to solve the problem.

5. Integer Programming and Scheduling (7p.)

A) **Scheduling (1p.)**

A paint factory has **one (1)** mixer on which **three (3)** different colors are produced: Blue, Green, and Yellow. Formulate the scheduling problem (**MILP**), where **two (2)** batches of each of the **three (3)** colors must be produced. The production time for each batch is: Blue: **2** *h*, Green: **3** *h*, Yellow: **4** *h*. Cleaning time between the batches depends on the previous color according to the table given below:

From / To	Blue	Green	Yellow
Blue	0	2	4
Green	1	0	2
Yellow	1	1	0

Please **formulate** the problem minimizing the make span (total production time). **Do not solve it!**

B) **Integer Programming (6p.)**

The owner of a manufacturing company is planning to purchase **two** (2) types of machines, namely Heater and Blender. He has estimated that purchasing each Heater will increase the profit by **$100** per day and each Blender machine will increase the profit by **$150** per day. The number of machines the owner can purchase is limited by the purchasing cost of the machines and the amount of floor space available. The following table shows the purchase prices of the machines and the space requirements for each machine. The owner has a budget of **$40000** for purchasing the machines and **200** square feet (ft^2) of available floor space. The owner wants to know how many units of each type of machine to purchase in order to maximize the daily increase in profit.

Machine	Required Floor Space (ft^2)	Purchase Price
Heater	15	$8000
Blender	30	$4000

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a. Formulate the problem as an integer programming problem.
b. Solve it graphically as a linear programming (LP) model without integer restrictions (relaxed problem).
c. Perform the Branch & Bound (B&B) algorithm graphically and compare the solutions.
d. How big is the integer gap?