1:	2:	3:	Extra:	**Total 1-3:**	**/ 25**
4:	5:			**Total 4-5:**	**/ 18**

Aalto ME-C3100 Computer Graphics, Fall 2017

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Tentti/Exam — Midterm 2/Välikoe 2, December 20 2017

Name, student ID: ________________________________

Allowed: Two two-sided A4 note sheets, calculators (also symbolic). Turn your notes in with your answers. **Write your answers in either Finnish or English on this sheet.**

The following applies to students who are taking the class in fall 2017: If you have not taken the midterm (välikoe) in October, you must answer *all* questions. Questions 1-3 cover the material from the 2nd half of class after October's midterm. Questions 4-6 cover the material from the first midterm. Only answer 4-6 if 1) you didn't take the midterm, or 2) wish to raise your midterm score. You're guaranteed not to make yourself worse off if you try: we'll scale the points accordingly and count only the better result.

1 Rendering Basics [/ 6]

1.1 Ray Casting vs. Rasterization [/ 4]

Give pseudocode for rendering an image using a ray caster and a rasterizer.

Ray Casting | Rasterization

1.2 Ray Tracer vs. Rasterizer Working Sets [/ 2]

What are the main differences between the working sets in rasterization and ray casting? For both algorithms:
a) what data needs to be kept in memory for random access during rendering? [/ 1]

b) what data can be processed in a stream (= computed once and then forgotten)? [/ 1]

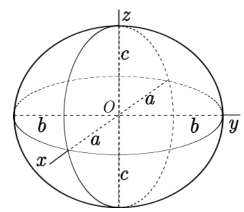
2 Ray Casting/Tracing and Rasterization [/ 12]

A ray with origin $\mathbf{o} = (o_x, o_y, o_z)$ and direction $\mathbf{d} = (d_x, d_y, d_z)$ is represented by $\mathbf{p} = \mathbf{o} + t\mathbf{d}$ in world coordinates.

2.1 Transforming Rays [/ 2]

A rigid object has a 4×4 object-to-world affine transformation matrix $\mathbf{M}$. Transform the world space ray $(\mathbf{o}, \mathbf{d})$ from to the object space ray $(\mathbf{o}', \mathbf{d}')$. Give the formulae for $\mathbf{o}'$, $\mathbf{d}'$ in terms of $\mathbf{M}, \mathbf{o}, \mathbf{d}$. Pay attention to what you do with the homogeneous coordinates! Do not normalize the resulting vectors.

2.2 Ray-Ellipsoid Intersection [/ 5]



An axis-aligned ellipsoid centered at the origin is defined by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where $a, b, c > 0$ are constants.

a) What is special about the ellipsoid when $a = b = c$? [**/ 1**]

b) Derive the formula for intersecting a ray with the ellipsoid. The result is a quadratic equation in t with coefficients A, B, C. Give the expressions for A, B, C. You *do not* need to solve for the actual intersections given A, B, C. *It is sufficient, and much easier, to write the solution in terms of matrices and vectors instead of writing out all the components. You can write the problem down in the form* $\mathbf{p}(t)^T\mathbf{M}\mathbf{p}(t) = 1$, *where* $\mathbf{p}(t)$ *is the vector expression for the ray in terms of its origin and direction and* $\mathbf{M}$ *is a particular matrix (that you need to derive), and take it from there.* [**/ 4**]

2.3 Rasterization: Edge Functions [/ 1]

Edge functions $e_i(x, y) = a_i x + b_i y + c_i$ are 2D line equations that are computed from the three edges ($i = 1, 2, 3$) of a projected triangle. What is the mathematical condition that holds when a pixel/sample at (x, y) is inside the triangle?

2.4 Edge Functions [/ 4]

Given two projected vertices (x_1, y_1) and (x_2, y_2), derive the coefficients a, b, c for an edge function $e(x, y) = ax + by + c$ that separates the plane into two half-spaces such that $e(x, y) = 0$ when the point (x, y) lies on the line defined by the two vertices, and has a positive value on one side (and resp. a negative value on the other side). It doesn't matter which side you choose to be positive. Note that the result is *not* unique: any positive non-zero multiple of a, b, c will give the same classification. Your solution will have to work for *all* lines, also purely vertical. *Hint: Write down the implicit line equation, and note that 1) its direction and normal are related in a simple way, and 2) a 90 degree plane rotation is almost a simple swap operation.*

3 Shading and Sampling [/ 7]

3.1 The BRDF [/ 1]

The BRDF stands for "Bidirectional Reflectance Distribution Function". It is often denoted by $f_r(\mathbf{l}, \mathbf{v})$, where $\mathbf{l}$ is incident (light) direction and $\mathbf{v}$ is the outgoing (viewing) direction. What does the value $f_r(\mathbf{l}, \mathbf{v})$ tell you, in simple intuitive terms?

3.2 Shadow Rays [/ 1]

What is special about shadow rays, as opposed to rays cast from the camera, in terms of intersection computations?

3.3 Aliasing [/ 4]

a) What is meant by pre-aliasing and post-aliasing? [/ **2**]

b) How does multisampling differ from supersampling? Why is it useful? [/ **2**]

3.4 Texturing [/ 1]

What is the name of the technique used for avoiding aliasing in texture mapping?

4 Linear Algebra [/ 7]

4.1 Linearity; Translation is not linear [/ 4]

Give the definition for a linear operation. Then, using your definition, show that translation $T_p(x) \mapsto x + p$ is *not* a linear operation in $\mathbb{R}^n$.

4.2 Affine Transforms [/ 3]

What happens to lines under a linear transformation? What about a pair of parallel lines? How is the effect of an affine transform different, if at all? (See extra credit.)

5 Dynamics [/ 11]

5.1 Particle modeling [/ 6]

Consider a 2D coordinate system $\mathbf{x} = (x, h)$, where x measures distance along the ground and h is height above ground. Consider a single, small but heavy mass thrown upwards from the origin $(0, 0)$ at initial velocity $\mathbf{x}'(0) = \mathbf{v}_0 = (x'_0, h'_0)$. (The prime denotes differentiation with respect to time t.) Newton's second law $\mathbf{F} = m\mathbf{a}$ governs the motion of the mass. The mass m is assumed to be so small we can model it as a particle. (See extra credit.)

a) The *phase space* describes all possible states of a physical system. What is the phase space $\mathbf{X}$ of this single-particle system? How many dimensions does it have? [/ **2**]

b) Rewrite the 2nd-order equation of motion as a system of first-order differential equations, including the explicit formula for the gravitational force (force pulling down, with constant of acceleration g). Denoting a point in phase space by $\mathbf{P}$, give an explicit formula for $\mathbf{P}' = \frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t}$. [/ **2**]

c) What is the state of the system after one timestep of Euler integration with step size h?
[/ **2**]

5.2 Numerical Integration of ODEs [/ 5]

a) What do you hope to accomplish when you use stiff springs for modeling cloth? [/ **1**]

b) The explicit Euler method has problems with stiff springs. What is the typical manifestation of this?
[/ **1**]

c) Why does it make sense to write higher-order ODEs as first-order systems? Give one reason.
[/ **1**]

d) What does the term *order* mean when talking about a numerical ODE solver? In big-Oh notation, how does the error E of an nth order solver behave with the step size h if the length of the simulated time interval is kept fixed? Also write out in words what this means for a 2nd order method. [/ **2**]

A Extra Credit

No partial credit for extra credit questions, except in cases where you make a clear technical mistake in solving an otherwise correctly-derived equation. You can also use the next blank page.

A.1 A strange transformation [/ 2]

What is the geometric action ("what does it do") of the 2D linear transformation encoded by the matrix

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}?$$

A.2 Lines under affine transformations [/ 4]

Prove that straight lines remain straight lines under affine transformations.

A.3 Analytic dynamics [/ 4]

Derive an analytic solution to the system from Problem 5.1.

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