

- (b) Give the null hypothesis of the test. (1 p.)
- (c) Assume that the ϵ_i are normally distributed. What is the distribution of the test statistic under the null hypothesis? (1 p.)
- (d) Assume that the ϵ_i are not normally distributed. Explain how, in this case, you can estimate the p -value of the test statistic. (2 p.)

3. Stationarity (6 p.)

Let ϵ_t be iid and assume that $E[\epsilon_t] = 0$ and $E[(\epsilon_t)^2] = \sigma^2, \sigma^2 < \infty$. Let $x_0 = \epsilon_0$ and let $x_t = x_{t-1} + \epsilon_t, t > 0$. Let D denote the difference operator. Show that the process $y_t = D^2x_t$ is weakly stationary.

4. Interval bootstrapping/block bootstrapping (6 p.)

Assume that you have observed a time series $x_1, x_2, x_3, \dots, x_{10263}$. Based on plotting the series and its estimated autocorrelation and partial autocorrelation -functions, you think that the observed series is a stationary AR(2) process

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \epsilon_t, \quad (\epsilon_t)_{t \in T} \sim WN(0, \sigma^2)$$

and you have estimated the parameters ϕ_1 and ϕ_2 .

- (a) Explain, step by step, how to construct a 95% bootstrap confidence intervals for the parameters ϕ_1 and ϕ_2 . (4 p.)
- (b) Assume that 0 is in the confidence interval that corresponds to ϕ_1 , but it is not in the confidence interval that corresponds to ϕ_2 . How would you interpret that? (2 p.)

5. Figures 1 and 2 display the theoretical autocorrelation and partial autocorrelation -functions of six different processes. Answer to the following questions. You do not have to justify your answers. (Every correct answer +1 p., every wrong answer 0 p., no answer 0 p.)

- (a) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(2)-process?
- (b) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a MA(1)-process?
- (c) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(1)-process?
- (d) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a SAR(2)₆-process?