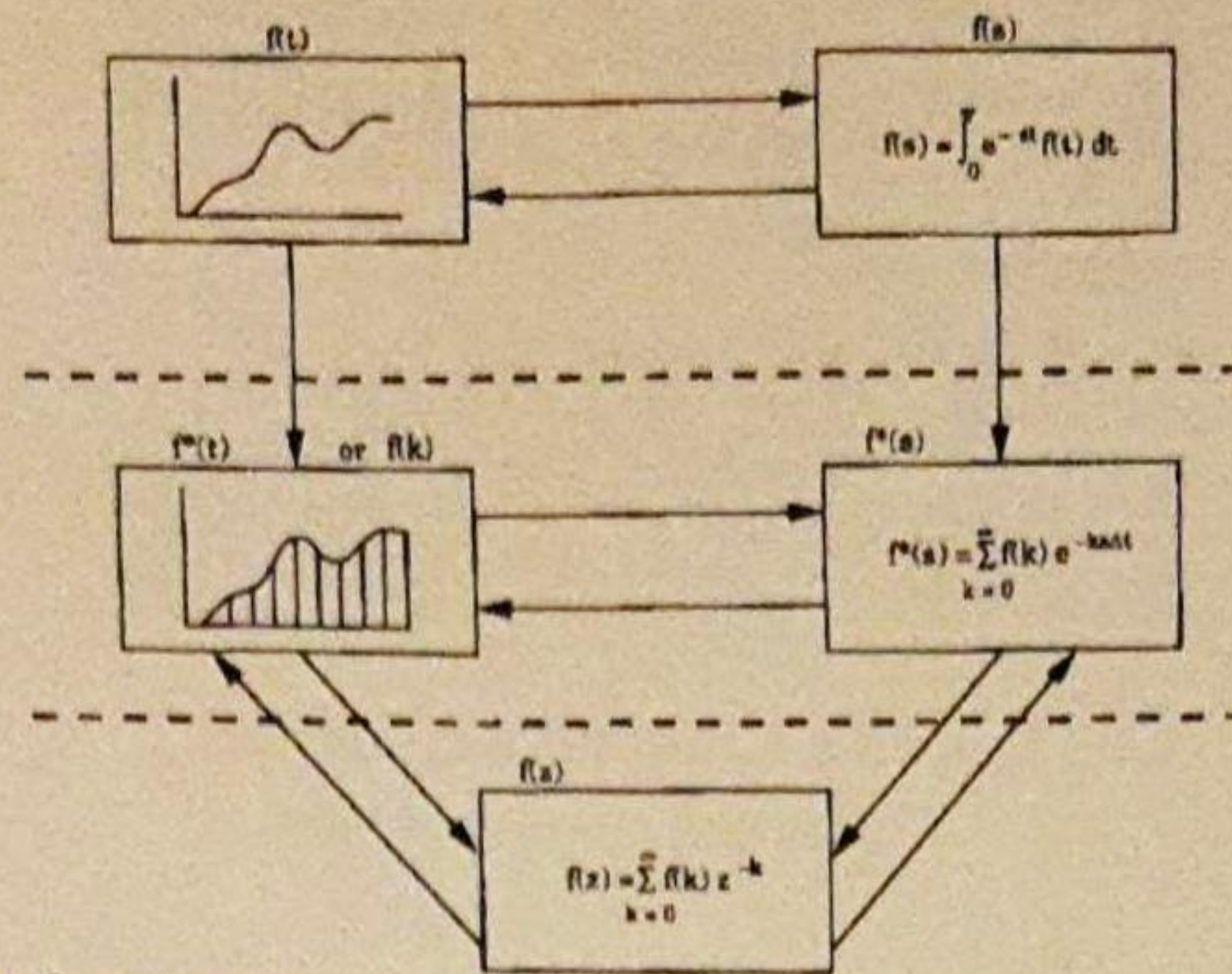


CHEM-E7165 Advanced Process Control Methods

Exam, 18.02.2019

1. Transformations

- a. Describe which are the transformations relating the continuous domain to the z-domain via the discrete domain



- b. The system transfer function is:

$$G(s) = \frac{1}{s(s+1)}$$

Make a state-space presentation of the $G(s)$, and discretize. The sampling interval is T

- c. Transform $G(s)$ into a pulse transfer function $H(z)$ for a discrete system

2. Digital control

- a. Explain the differences between continuous time and discrete time PID-controller (Explain also tuning)

b. Given a system:

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ -0.25 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(k)$$

Is it possible to find controls $u(0)$ and $u(1)$ so that $x(2) = [-0.5 \ 1]^T$ with given initial point $x(0) = [2 \ 2]^T$?

3. Model Predictive Control

Explain the purpose of each of the terms in the following general form of the constrained MPC problem:

$$\min_{(\Delta u(k), \dots, \Delta u(k+m-1))} \sum_{l=1}^p \varepsilon_r(k+l)^T \Gamma \varepsilon_r(k+l) + \sum_{l=1}^m u(k+l-1)^T F u(k+l-1) + \sum_{l=1}^{m-1} \Delta u(k+l-1)^T R \Delta u(k+l-1)$$

4. Describe the main differences between the following identification methods:

- Least Squares
- Generalized Least Squares
- Extended Least Squares
- Maximum Likelihood
- Instrumental Variable

5. Multivariable control

Design decouplers for the following system using both simplified and generalized decoupling approaches. Compare the decouplers and the resulting open loop transfer functions.

$$G = \begin{bmatrix} \frac{12.8}{16.7s+1} & \frac{-18.9}{21.0s+1} \\ \frac{6.6}{10.9s+1} & \frac{-19.4}{14.4s+1} \end{bmatrix}$$

6. Recursive least squares method

The following data was obtained from a process:

T	1	2	3	4	5	6	7	8	9
u(t)	1.0	2.0	0.5	-1.0	-0.4	1.0	0.6	2.0	1.3
y(t)	3.34	2.17	3.88	6.57	5.44	1.25	-0.78	1.72	2.61

Identify the model parameters using the RLS method:

$$y(k) = Ay(k-1) + Bu(k-1),$$

$$P(k) = P(k-1) - \frac{P(k-1)\varphi(k)\varphi(k)^T P(k-1)}{1 + \varphi(k)^T P(k-1)\varphi(k)}$$

$$\varepsilon(k) = y(k) - \varphi(k)^T \hat{\theta}(k-1)$$

$$K(k) = P(k)\varphi(k)$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)\varepsilon(k)$$

7. Kalman Filter

A process is described by the following state space model

$$x_{k+1} = Ax_k + Bu_k + w_k$$

$$y_k = Cx_k + v_k$$

where $A = \begin{bmatrix} 0.84 & -0.28 & 0.37 \\ -0.41 & -0.68 & 0.48 \\ 0.17 & -0.60 & 0.37 \end{bmatrix}$, $B = \begin{bmatrix} -0.1 \\ 0.7 \\ 0.3 \end{bmatrix}$, $C = \begin{bmatrix} 0.5 & -1.0 & 0.1 \\ 1.5 & -0.4 & -1.1 \end{bmatrix}$,

$R_w = \begin{bmatrix} 0.0025 & 0 & 0 \\ 0 & 0.0025 & 0 \\ 0 & 0 & 0.0025 \end{bmatrix}$, $R_v = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$, and the Kalman gain matrix $K =$

$\begin{bmatrix} 0.10 & 0.28 \\ -0.25 & -0.07 \\ 0.05 & -0.11 \end{bmatrix}$ if the state estimation $\hat{x}_0 = \begin{bmatrix} 0.01 \\ -0.12 \\ -0.11 \end{bmatrix}$, the input sequence $u_0 =$

$-0.3, u_1 = 1.3, u_2 = 0.5$, and the measured outputs $y_1 = \begin{bmatrix} 0.0851 \\ -0.2241 \end{bmatrix}$, $y_2 = \begin{bmatrix} -0.9164 \\ 0.3827 \end{bmatrix}$

Calculate the state estimations $\hat{x}_{1|1}$ and $\hat{x}_{2|2}$ using the Kalman filter!

$$K_k = P_{k|k-1}C^T(CP_{k|k-1}C^T + R_v)^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - C\hat{x}_{k|k-1}) \quad \text{Correction}$$

$$P_{k|k} = (I - K_kC)P_{k|k-1}$$

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k$$

$$P_{k+1|k} = AP_{k|k}A^T + GR_wG^T \quad \text{Prediction}$$