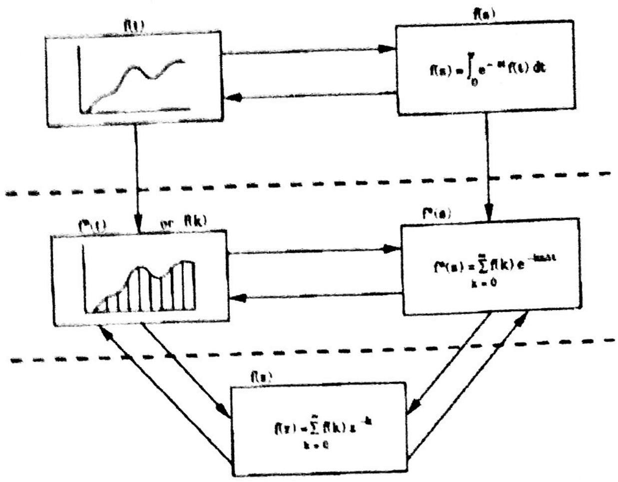
# CHEM-E7165 Advanced Process Control Methods
Exam, 18.02.2019

1. **Transformations**
   a. Describe which are the transformations relating the continuous domain to the z-domain via the discrete domain



   b. The system transfer function is:

$$G(s)=\frac{1}{s(s+1)}$$

   Make a state-space presentation of the $G(s)$, and discretize. The sampling interval is T

   c. Transform $G(s)$ into a pulse transfer function $H(z)$ for a discrete system

2. **Digital control**
   a. Explain the differences between continuous time and discrete time PID-controller (Explain also tuning)

   b. Given a system: $$x(k+1)=\begin{bmatrix}1 & 1\\ -0.25 & -1\end{bmatrix}x(k)+\begin{bmatrix}1\\ -0.5\end{bmatrix}u(k)$$

Is it possible to find controls $u(0)$ and $u(1)$ so that $x(2)=[-0.5 \quad 1]^T$ with given initial point $x(0)=[2 \quad 2]^T$ ?

3. **Model Predictive Control**

Explain the purpose of each of the terms in the following general form of the constrained MPC problem:

$$\min_{\{\Delta u(k),\ldots,\Delta u(k+m-1)\}} \sum_{l=1}^{p}\varepsilon_y(k+l)^T\,\Gamma\,\varepsilon_y(k+l)+\sum_{l=1}^{m}u(k+l-1)^T\,Fu(k+l-1)$$
$$+\sum_{l=1}^{m-1}\Delta u(k+l-1)^T\,R\Delta u(k+l-1)$$

4. **Describe the main differences between the following identification methods:**
   a. Least Squares
   b. Generalized Least Squares
   c. Extended Least Squares
   d. Maximum Likelihood
   e. Instrumental Variable

**5. Multivariable control**

Design decouplers for the following system using both simplified and generalized decoupling approaches. Compare the decouplers and the resulting open loop transfer functions.

$$G = \begin{bmatrix} \dfrac{12.8}{16.7s+1} & \dfrac{-18.9}{21.0s+1} \\ \dfrac{6.6}{10.9s+1} & \dfrac{-19.4}{14.4s+1} \end{bmatrix}$$

**6. Recursive least squares method**

The following data was obtained from a process:

| T | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| u(t) | 1.0 | 2.0 | 0.5 | -1.0 | -0.4 | 1.0 | 0.6 | 2.0 | 1.3 |
| y(t) | 3.34 | 2.17 | 3.88 | 6.57 | 5.44 | 1.25 | -0.78 | 1.72 | 2.61 |

Identify the model parameters using the RLS method:

y(k) = Ay(k-1) + Bu(k-1),

$$P(k) = P(k-1) - \frac{P(k-1)\varphi(k)\varphi(k)^T P(k-1)}{1+\varphi(k)^T P(k-1)\varphi(k)}$$
$$\varepsilon(k) = y(k) - \varphi(k)^T\hat{\theta}(k-1)$$
$$K(k) = P(k)\varphi(k)$$
$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)\varepsilon(k)$$

**7. Kalman Filter**

A process is described by the following state space model

$$x_{k+1} = Ax_k + Bu_k + w_k$$
$$y_k = Cx_k + v_k$$

where $A = \begin{bmatrix} 0.84 & -0.28 & 0.37 \\ -0.41 & -0.68 & 0.48 \\ 0.17 & -0.60 & 0.37 \end{bmatrix}$, $B = \begin{bmatrix} -0.1 \\ 0.7 \\ 0.3 \end{bmatrix}$, $C = \begin{bmatrix} 0.5 & -1.0 & 0.1 \\ 1.5 & -0.4 & -1.1 \end{bmatrix}$,
$R_w = \begin{bmatrix} 0.0025 & 0 & 0 \\ 0 & 0.0025 & 0 \\ 0 & 0 & 0.0025 \end{bmatrix}$, $R_v = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$, and the Kalman gain matrix $K =$
$\begin{bmatrix} 0.10 & 0.28 \\ -0.25 & -0.07 \\ 0.05 & -0.11 \end{bmatrix}$ if the state estimation $\hat{x}_0 = \begin{bmatrix} 0.01 \\ -0.12 \\ -0.11 \end{bmatrix}$, the input sequence $u_0 =$
$-0.3, u_1 = 1.3, u_2 = 0.5$, and the measured outputs $y_1 = \begin{bmatrix} 0.0851 \\ -0.2241 \end{bmatrix}, y_2 = \begin{bmatrix} -0.9164 \\ 0.3827 \end{bmatrix}$

**Calculate the state estimations $\hat{x}_1$ and $\hat{x}_2$ using the Kalman filter!**

$$\begin{aligned} K_k &= P_{k|k-1}C^T(CP_{k|k-1}C^T + R_v)^{-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k(y_k - C\hat{x}_{k|k-1}) \\ P_{k|k} &= (I - K_kC)P_{k|k-1} \end{aligned}$$

**Correction**

$$\begin{aligned} \hat{x}_{k+1|k} &= A\hat{x}_{k|k} + Bu_k \\ P_{k+1|k} &= AP_{k|k}A^T + GR_wG^T \end{aligned}$$

**Prediction**