

Midterm I, 20.2.2019

1. Why can the baseband signal in carrier modulation be expressed as a complex-valued signal, while the true signal is a real-valued waveform, where the amplitude of the carrier wave is modulated? What does this mean if BPSK modulation is used? How about QAM modulation? How does the bandwidth of the baseband signal relate to the bandwidth of the passband signal?
2. One bit is transmitted by selecting one of two signal waveforms x_0 and x_1 which are created from two basic waveforms s_1 and s_2 , see Fig. 1. The binary input data is mapped such that

$$\begin{aligned} 0 \rightarrow x_0(t) &= \frac{1}{2}s_1(t) - \frac{1}{2}s_2(t) \\ 1 \rightarrow x_1(t) &= -\frac{1}{2}s_1(t) + \frac{1}{2}s_2(t) \end{aligned}$$

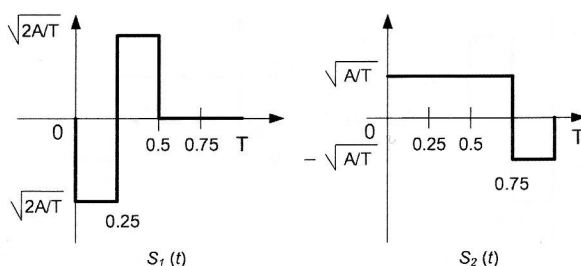
The received signal is $r(t) = x_i(t) + n(t)$ where $n(t)$ is additive Gaussian noise with standard deviation $\sigma^2 = N_0/2$.

- a) Are the basic waveforms s_1 and s_2 orthogonal?
- b) Are the waveforms x_0 and x_1 orthogonal?
- c) Is it possible to interpret the transmission in terms of BPSK symbols?
- d) Derive the optimum receiver (matched filter + decision boundary). Hint: After properly normalized matched filtering, the probability density of the received signal samples r_i are

$$p(r_i|x_i) = \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(r_i - x_i)^2}{N_0}\right)$$

where x_i is the filtered transmission waveform.

3. Let us take the real-valued random variable x , which is uniformly distributed in $[-1/2, 1/2]$. We quantize this variable to equal-size quantization bins, such that the quantization levels are midpoints of quantization intervals. Compute the quantization signal-to-noise ratio as a function of the number of bits. How many bits are needed to reach a SNR of at least $\gamma_q = 15\text{dB}$? Hint: The expected power of the quantization distortion noise is $D = E\{(x - \hat{x})^2\}$, while the expected signal power is $S = E\{x^2\}$.



Kuva 1: The signals s_1 and s_2 , A is the square root of the energy and T is the duration of the signal