Aalto University School of Electrical Engineering
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EXAM
20.2.2019

ELEC-E7240 Coding Methods

All answers must be well motivated.

1. (6p.) Algebra. Consider the ring $R_4 = \mathrm{GF}(2)[x]/(x^4+1)$
   (a) (2p.) How many elements are there in the ring R_4? Justify your answer.
   (b) (2p.) Multiply the polynomials x^3+x^2+1 and x^2+x in R_4. *Hint:* The answer will have degree less than 4.
   (c) (2p.) An *ideal* in a commutative ring $(R,+,\cdot)$ is a subset I of the elements of R with the property that
   - $(I,+)$ is a subgroup of $(R,+)$ and
   - for every $r \in R$ and every $x \in I$, the product $r \cdot x$ is in I.

   Find an ideal in the ring R_4 which consists neither of all elements of the ring nor of just the additive identity (0). *Hint 1:* The smallest such ideal has only two elements. *Hint 2:* If you want to gain some understanding of what is going on, consider ideals in the ring of addition and multiplication modulo 6.

2. (6p.) Block codes. A (7,4) Hamming code has the following set of codewords:

0000000	1101000	0110100	1011100
0011010	1110010	0101110	1000110
0001101	1100101	0111001	1010001
0010111	1111111	0100011	1001011

   (a) (2p) Construct a parity check matrix for this code.
   (b) (2p) Draw a Tanner graph corresponding to the parity-check matrix obtained in part (a).
   (c) (2p) This code is also cyclic. Give a generator polynomial of this code.

3. (6p.) Coding methods.
   (a) (2p.) What is an interleaver?
   (b) (4p.) Describe the principles of turbo coding.

4. (6p.) Convolutional codes. Consider the convolutional code defined by the transfer-function matrix

$$\mathbf{G}(D) = \begin{bmatrix} 1+D & 1+D^2 & 1+D+D^2 \end{bmatrix}.$$

   (a) (2p.) Draw a picture (shift-register circuit) of an encoder of this code.
   (b) (4p.) This code is used for transmission over a binary symmetric channel. The received sequence is 010 111 110 110 010 110 011. Use the Viterbi algorithm for decoding to find both (1) the sequence that was transmitted across the channel and (2) the original message.