

**FINAL EXAM,  
FIRST COURSE IN PROBABILITY AND STATISTICS**

- **Time:** 20.2.2019, 9:00-12:00
- **Equipment:** Calculator and one sheet (A4) of hand-written notes, written on one side only.
- Answer each problem on a separate page. Each problem is worth 6 points.
- Motivate all solutions carefully. Answers without motivation give no points.
- Mark your course code on the front page.

PROBLEM 1

A red, white and blue die are rolled (all three dice are fair and six-sided). Denote their outcomes respectively by  $A$  (red die),  $B$  (white die) and  $C$  (blue die).

- Compute the conditional probability  $P(A < C|A = i)$ ,  $i = 1, \dots, 6$ . (1p)
- Compute the probability  $P(A < C)$ . (1p)
- Compute the conditional probability

$$P(\{A < B\} \cap \{A < C\}|A = i), \quad i = 1, \dots, 6. \quad (1p)$$

- Compute the joint probability  $P(\{A < B\} \cap \{A < C\})$ . (1p)
- Compute the conditional probability  $P(A < B|A < C)$ . (2p)

PROBLEM 2

60% of the Finnish population is “young”, by which we mean below 50 years, and the rest is “old”. 35% of young people use glasses, whereas 85% of old people do.

A sample of 100 individuals are selected at random (with replacement). Let  $X$  be the number of young people in the sample, and let  $Y$  be the number of people in the sample who wear glasses.

- Compute  $E(X)$ . (1p)
  - Compute  $E(Y)$ . (2p)
  - Compute the covariance  $\text{Cov}(X, Y)$ . (3p)
- (hint: write  $X$  and  $Y$  as sums of indicator variables.)

PROBLEM 3

100 random numbers are drawn independently from the continuous uniform distribution on  $[-1, 2]$ . Let  $X$  be the number of positive numbers drawn. Use the normal approximation to estimate  $P(X < 60)$ . (6p)

PROBLEM 4

The time  $X$  (in seconds) from when I leave my office until I jump on the metro can be modelled as a constant time  $c$  (to walk to the metro station) plus an exponentially distributed time with rate  $\lambda$  (waiting). So the probability density function of  $X$  is

$$f(t) = \begin{cases} \lambda e^{-\lambda(t-c)}, & t \geq c \\ 0, & \text{otherwise} \end{cases}$$

The waiting times on different days are supposed to be independent. The last five days,  $X$  was 185, 400, 250, 500, 375.

- Write down the likelihood function for the unknown parameters  $c$  and  $\lambda$ . (2p)
- Compute the maximum likelihood estimate of  $c$ . (2p)
- Compute the maximum likelihood estimate of  $\lambda$ . (2p)

## 1. STATISTICAL TABLES

Kertymäfunktion  $\Phi(z)$  arvoja / Values of the cumulative distribution function  $\Phi(z)$