TENTTI / TENT / EXAM

Kurssikoodi / Kurskod / Course code	MS-E2148	Kurssin nimi / Kursnamn / Course name	Dynamic optimization
Tentin päivämäärä / Tent datum / Exam date	9.4.2019	Tentin kesto / Provtid / Exam duration	3 H
Vastuuopettaja / Ansvarig lärare / Responsible teacher	Harri Ehtamo	Tenttipaperin kieli(kielet) / Tentpapperets språk / Language(s) of the exam paper	SUOMI
☐ Välikoe / Mellanprov / Midterm exam		☒ Tentti / Tent / Final exam	

SALLITUT APUVÄLINEET	TILLÅTNA HJÄLPMEDEL	ALLOWED MATERIAL
Tentissä saa käyttää vain kynää, kumia, viivainta ja harppia sekä niitä apuvälineitä jotka ovat merkitty sallituiksi oheiseen listaan.	I tenten får man använda enbart penna, radergummi, linjal och passare samt hjälpmedel som är markerat som tillåtet i listan nedan.	In the exam it is only allowed to use a pen, eraser, ruler and compass and the material that is marked as allowed in the list below.

	Sallittu Tillåten Allowed	Kielletty Förbjuden Forbidden
Funktiolaskin / Funktionsräknare / Regular calculator	☒	☐
Graafinen laskin (tyhjennettävä) / Grafräknare (bör tömmas) / Graphical calculator (to be emptied)	☒	☐
Laininen: kaava- ja taulukkokokoelma (valvojat jakavat) / formel och tabellsamling / formulary and table compendium	☐	☒
Mellin: kaava- ja taulukkokokoelma (opiskelijat tuovat itse) / formel och tabellsamling / formulary and table compendium	☐	☒
Muu materiaali (materiaali eriteltynä ja ohjeet sen tarkastukseen alla) / Övrig material (nedan) / Other material (below)	☐	☒

MUUT OHJEET / ÖVRIGA ANVISNINGAR / OTHER INSTRUCTIONS

☐	Tenttipaperi on palautettava valvojille Tentpapperet bör inlämnas åt övervakarna Exam paper must be returned	☒	Tenttipaperin saa viedä mukanaan Tentpapperet får tas med The exam paper can be taken

Kirjoita selvällä käsialalla jokaiseen vastauslomakkeeseesi tentin päivämäärä ja sali, kurssikoodi ja -nimi, opiskelijanumerosi, nimesi, koulutusohjelmasi ja allekirjoituksesi.

Jätä apuvälineet joita ei tarvita tentissä (kuten kännykät tai muut elektroniset laitteet) salin sivuille tai valvojille.

Varmista että nimesi kirjataan osallistujalistaan palauttaessasi vastauksesi. Tämä todistaa palautuksesi mikäli vastauslomakkeesi katoavat.

Suttupaperit merkitään kirjoittamalla paperiin suurilla kirjaimilla sana "SUTTU" ja vetämällä henkselit kirjoitusta sisältävien sivujen yli. Suttupapereita ei arvostella.

Tentissä noudatetaan yliopiston tenttiohjesääntöä.
Tenttikysymykset alkavat seuraavalta sivulta. KÄÄNNÄ SIVUA VAIN LUVALLA!

Skriv tydligt i varje svarsblankett tentens datum och sal, kurskod och -namn, samt ditt studienummer, namn, utbildningsprogram och underskrift.

Lämna de hjälpmedel som inte behövs i tenten (bl.a. mobiltelefoner och övriga elektroniska apparater) vid salens sidor eller åt övervakarna.

Försäkra att ditt namn skrivs på deltagarlistan när du inlämnar din prestation. Det bevisar din inlämning ifall dina svarsblanketter försvinner.

Klotterpapperen markeras med att skriva med stora bokstäver ordet "SUTTU" och med att dra ett kryss över varje sida som innehåller skrift. Klotterpapperen bedöms ej.

I tenten följer man universitetets tentamensregler.

Tentfrågorna börjar på nästa sida. VÄND SIDA ENBART MED TILLSTÅND!

Write clearly on every answer sheet the exam date and hall, course code and name, and your study number, name, study programme and signature.

Leave material that is not needed in the exam (e.g. mobile phones or other electronical devices) on the sides of the hall or to the supervisors.

Ensure that your name is written in the list of participants when returning your answers. It proves the returning if your answer sheets get lost.

Scratch papers are marked with the word "SUTTU" in capital letters and by drawing a large cross over all pages that have writing on them. Scratch papers are not graded.

The university's exam regulations are followed in the exam.

Exam questions start on the next page. TURN PAGE ONLY WITH PERMISSION!

Exam 9.4.2019

1. a) Starting from the general transversality condition in optimal control,

$$[h_x(x^*(t_f),t_f) - p^*(t_f)]^T \delta x_f + [H(x^*(t_f),u^*(t_f),p^*(t_f),t_f) + h_t(x^*(t_f),t_f)]\delta t_f = 0,$$

justify the different optimal control end point conditions given in the appendix below.

b) Suppose the Hamiltonian does not explicitly depend on time, and suppose a fixed final time problem. Show that $(d/dt)H(x^*(t),u^*(t),p^*(t)) = 0$ for all t.

2. Solve the optimal control in feedback form to the problem of minimizing

$$J = \frac{1}{4}x^2(T) + \int_0^T \frac{1}{4}u^2(t)dt,$$

subject to $\dot{x} = x(t)+u(t)$. Hint: Use the HJB and solve it using the trial $J(x,t) = \frac{1}{2}K(t)x^2$ for the optimal cost-to-go. Solve also $K(t)$ explicitly.

3. Explain or describe shortly six of the following 11 concepts:
   a. functional; give an example
   b. autonomous problem in the calculus of variation
   c. switching function
   d. Pontryagin minimum principle
   e. stochastic discrete time dynamic system
   f. additive cost functional
   g. the principle of optimality; give an example
   h. cost-to-go at stage i and state x_i (stochastic optimal control)
   i. shortest path problem
   j. optimal cost-to-go (continuous time optimal control)
   k. Hamilton-Jacobi-Bellman equation (continuous time optimal control).

4. You are participating in a coin-tossing game where you win 2$ whenever heads occurs and lose 1$ whenever tails occurs. However, you suspect that the coin is biased, i.e., heads and tails do not occur with equal probability. You observed your friend playing the game before, and he got z heads on n tosses. You are allowed a maximum of N tossings. Thus, your problem is to find an optimal policy of deciding whether to continue or stop participating in the game given the outcomes of the game so far. Indicate how such a policy could be found, by formulating the DP-algorithm for this problem.

5. Your task is to to find the optimal alignment for the two DNA-sequences:

   (1) G A A T T C A G T T A

   (2) G G A T C G A

   To find out similarity of the sequences, we weight different alignments. $S_{i,j} = 1$ if base in sequence 1 at place j matches the base of sequence 2 at place i. We also assume that in evolution there are two types of mutations that happen more than other; C $\rightarrow$ T and T $\rightarrow$ C. Thus, also $S_{i,j} = 1$ if in sequence 1 in the place j is the structural unit T or C and in the sequence 2 in the place i is the structural unit C or T. In other cases, $S_{i,j} = 0$. There can also be gaps in sequence 1 or 2, which is penalized by $w = 0$.

   The task is to find the alignment with highest score. We design an alignment matrix whose elements are:

$$M_{i,j} = \max \left\{ \begin{array}{ll} M_{i-1,j-1} + S_{i,j}, & \text{(match/no match)} \\ M_{i,j-1} + w, & \text{(gap in sequence 2)} \\ M_{i-1,j} + w, & \text{(gap in sequence 1)} \end{array} \right\}$$

   The text in the bracket gives the "control" how the sequence (2) is built. Also assume that $i, j = 1, 2, ...$ and $M_{0,0} = M_{i,0} = M_{0,j} = 0$

   Using the DP-algorithm, we get the following scoring matrix

	G	A	A	T	T	C	A	G	T	T	A
G	1	1	1	1	1	1	1	1	1	1	1
G	1	1	1	1	1	1	1	2	2	2	2
A	1	2	2	2	2	2	2	2	2	2	3
T	1	2	2	3	3	3	3	3	3	3	3
C	1	2	2	3	4	4	4	4	4	4	4
G	1	2	2	3	4	4	4	5	5	5	5
A	1	2	3	3	4	4	5	5	5	5	6

   Your task is to find all optimal alignments (i.e., which alignments produce the score 6 in the scoring matrix) by backtracking from the scoring matrix.

APPENDIX:
HJB: $0 = J_t + \min_{u(t)} \{g + J_x^T f\}$
E-L: $0 = g_x - \frac{d}{dt}(g_{\dot{x}})$
Hamiltonian: $H = g + p^T(t) f(x(t), u(t), t)$
costate: $\dot{p}(t) = -\frac{\partial H}{\partial x}$
free final state: $0 = g_{\dot{x}}$ or $h_x - p = 0$
free final time: $0 = g - g_{\dot{x}}\dot{x}$ or $H + h_t = 0$
free final state and time: $g_x = g_{\dot{x}} = 0$ or $h_x - p = 0 = H + h_t$
goal: $0 = g + \left[\frac{\partial g}{\partial \dot{x}}\right]^T \left[\frac{d\theta}{dt} - \dot{x}\right]$ or $H + h_t + (h_x - p)^T \frac{d\theta}{dt} = 0$
W-E: $g_{\dot{x}}$ and $g - g_{\dot{x}}\dot{x}$ continuous