Prediction and Time Series Analysis — 2018
Exam
Answer to all the questions.

You are allowed to have pens and pencils, an eraser and a ruler, and one size A4 note (handwritten, text on one side only, name on the top right corner).

1. True or False (6 p.)

   Determine whether the statement is true or false. You do not have to justify your answers. Simply state wether the statement is true or false. (Every correct answer +1 p., every wrong answer -1 p., no answer 0 p.)

   (a) In the context of linear regression, traditional least-squares estimators are sensitive to outlying observations.

   (b) In the context of linear regression, the variance inflation factors (VIF) are calculated in order to detect heteroscedasticity.

   (c) In time series analysis, differencing and seasonal differencing can be applied in order to stationarize the process.

   (d) The theoretical partial autocorrelation function of a moving average process of order 3 is equal to 0 after 3.

   (e) Autoregressive processes can be applied in predicting only if the residual terms of the process are normally distributed.

   (f) ARIMAX models have been designed for long term forecasting.

2. Linear regression (6 p.)

   Consider the linear regression model

   $$y_i = \beta_0 + \beta_1 x_{i1} + \epsilon_i.$$

   You have a sample and you estimate the parameters β_0 and β_1 using traditional least squares estimators. In your sample, you have two separate subgroups that have sample sizes h and $n - h$, and you are worried about possible parameter instability.

   (a) Explain, step by step, how to apply a permutation test in testing parameter instability. (4 p.)

   (b) Give the null hypothesis of the test. (1 p.)

(c) How would you proceed your modeling (what would you do), if the estimated p-value of the test statistic was very small? (1 p.)

3. Autocorrelation (6 p.)

   Let ρ_k denote the kth autocorrelation coefficient of a stationary stochastic process $(x_t)_{t\in T}$.

   (a) Prove that $\rho_0 = 1$. (1 p.)

   (b) Prove that $\rho_{-k} = \rho_k$ for all $k \in \mathbb{Z}$. (2 p.)

   (c) Prove that $|\rho_k| \leq 1$ for all $k \in \mathbb{Z}$. (3 p.)

4. ARMA modeling (6 p.)

   Assume that you have observed a series $x_1, x_2, x_3, ..., x_{5012}$.

   (a) Based on plotting the series, you observe a linear trend. You manage to stationarize the process by taking a difference. Give the elements of the obtained stationary process in terms of the elements of the original observed series. (1 p.)

   (b) Based on plotting the stationarized series and its estimated autocorrelation and partial autocorrelation -functions, you think that the observed series is a pure invertible moving average process of order 2. Give the definition of a moving average process of order 2. (2 p.)

   (c) You decide apply traditional ARMA-modeling based prediction to calculate the 1, 2, 3 and 4 step predictions for the stationarized series. What are the predicted values of x_{5013}, x_{5014}, x_{5014} and x_{5015}? (3 p.)

5. Figures 1 and 2 display the theoretical autocorrelation and partial autocorrelation -functions of six different processes. Answer to the following questions. You do not have to justify your answers. (Every correct answer +1 p., every wrong answer 0 p., no answer 0 p.)

   (a) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a MA(3)-process?

   (b) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(1)-process?

   (c) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(3)-process?

(d) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a $SMA(3)_3$-process?

(e) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a $SMA(3)_6$-process?

(f) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an ARMA(2,2)-process?

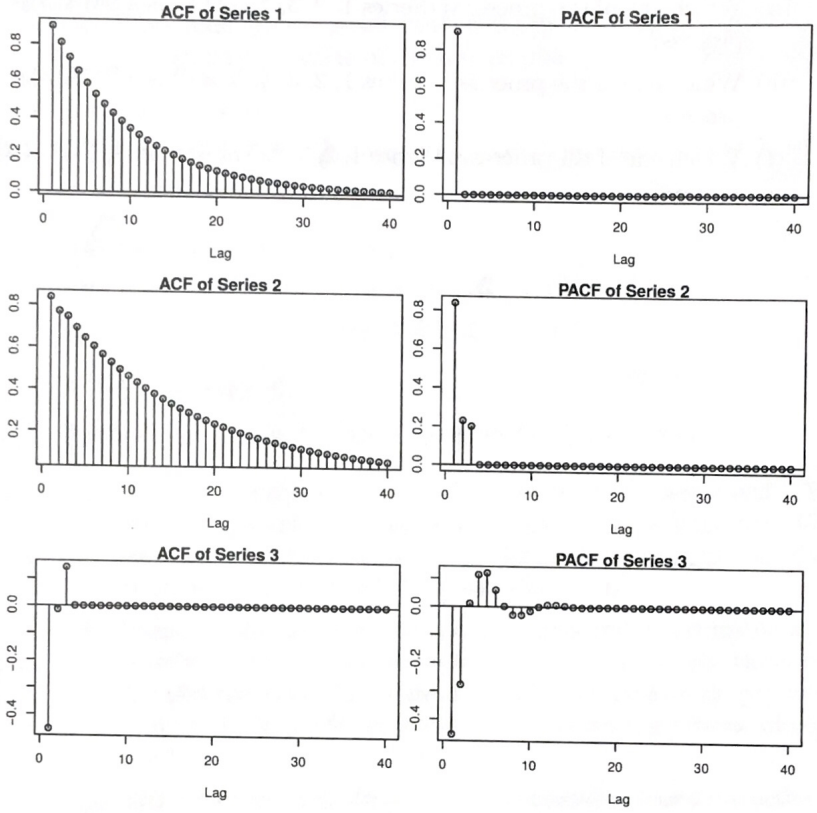
ACF of Series 1
PACF of Series 1
ACF of Series 2
PACF of Series 2
ACF of Series 3
PACF of Series 3
Lag

Figure 1

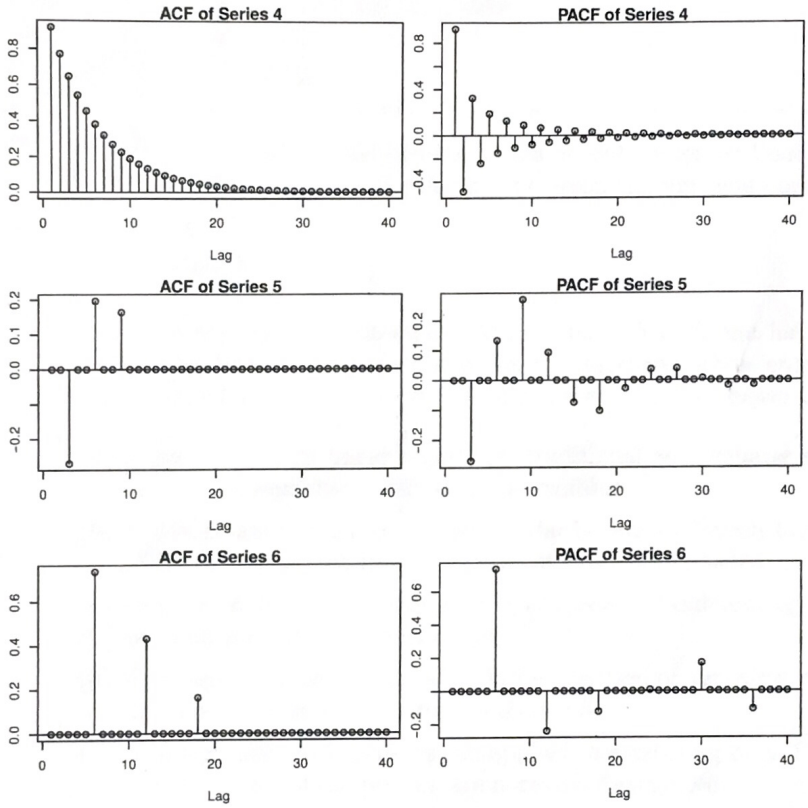
ACF of Series 4
PACF of Series 4
ACF of Series 5
PACF of Series 5
ACF of Series 6
PACF of Series 6
Lag

Figure 2