**Guidelines:** Write briefly and clearly, but justify your answers. A lone number as an answer does not yield points. The exam has 4 problems, each worth 0–6 points. Each answer sheet should contain:

- Course name and code
- LASTNAME and FIRSTNAMES (in block letters)
- Student number
- Study program and year
- Date and signature

**Allowed equipment:** Mellin's Statistical tables, calculator, a4 note card (hand written, text only on one side, own name in the upper right corner)

---

**T1** Four Mac users and two PC users were asked how often they rebooted their computers during last week. The Mac users answered: 1,2, 2, and 6 times. The PC users answered: 2 and 5 times. A random student was selected from among the set of students. Denote

$$T = \text{computer type of the random student (0=Mac, 1=PC)},$$
$$X = \text{number of times the computer of the random student was rebooted.}$$

(a) Write down a table that describes the joint distribution of $T$ and $X$. **(2p)**

(b) Compute the expected values of $T$ and $X$. **(1p)**

(c) Are $T$ and $X$ dependent or independent? Justify. **(1p)**

Afterwards one random Mac user and one random PC user were selected, independently of each other. Denote by $X_0$ and $X_1$ the numbers of times their computers were rebooted.

(d) Compute the probability $P(X_0 > X_1)$. **(1p)**

(e) Find out the correlation of $X_0$ and $X_1$. **(1p)**

**T2** A sceptics exam contains six problems, and each will be answered by selecting one of two alternatives. Each correct answer gives one point. Diligent and lazy students will participate in the exam. A diligent student has prepared well and will answer each question correctly with probability 0.9, whereas a lazy students answer uniformly randomly to each question. Based on course feedback, the teacher estimates that 2/3 of the students are diligent.

(a) What is the probability that a diligent student obtains precisely 3 points? **(2p)**

(b) What is the probability that a random student obtains precisely 3 points? **(2p)**

(c) What is the probability that a student who obtained precisely 3 points is lazy? **(2p)**

**T3** A data source is known to generate independent random numbers, normally distributed with mean $\mu$ (unknown) and standard deviation $\sigma = 2$. A set of $n = 100$ observations have been collected from the source. Test with 5% significance level the null hypothesis $H_0 : \mu = 3$ againts the alternative hypothesis $H_1 : \mu \neq 3$. Use the test statistic

$$t(x) \;=\; \frac{m(x) - 3}{\sigma/\sqrt{n}},$$

where $m(x) = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the mean value of the observed data set.

(a) Determine the p-value of the test statistic for a data set with mean 3.6. **(2p)**

(b) What is the conclusion of the test for the observed data in (a)? **(1p)**

(c) Determine the set of values of the test statistic which lead to rejecting the null hypothesis. **(1p)**

(d) Determine the probability of rejection error when the true value of the unknown parameter is known to be $\mu = 3.2$. **(2p)**

**T4** The waiting times (min) on different travel days on a bus stop are independent of each other, and follow the uniform distribution on the continuous interval $[0, \theta]$, with density function

$$f(t \,|\, \theta) \;=\; \begin{cases} \frac{1}{\theta}, & 0 < t < \theta, \\ 0, & \text{else}. \end{cases}$$

During five travel days, the waiting times $x = (1, 6, 3, 4, 8)$ have been observed. Help Ronald, Karl, and Thomas to estimate the value of $\theta$ based on these observations.

(a) Ronald chooses the maximum likelihood estimate. Compute this for the data set $x$. **(2p)**

(b) Karl's estimate is determined by solving $\theta$ from the equation

$$\mathrm{E}(X \,|\, \theta) \;=\; \frac{1}{n}\sum_{i=1}^{n} x_i,$$

where the left side is the mean of a random variable distributed according to $f(t \,|\, \theta)$, and the right side is the mean of the observed data. Compute this for the data set $x$. **(1p)**

(c) Explain what an unbiased estimator is. Is Karl's estimator unbiased? **(1p)**

(d) Thomas interprets the unknown parameter as a random variable $\Theta$ and chooses a prior distribution with density function

$$f_0(\theta) \;=\; \begin{cases} 2\theta^{-3}, & \theta \geq 1, \\ 0, & \text{else}. \end{cases}$$

Thomas chooses the posterior expectation of $\Theta$ as his estimate. Compute this estimate for the data set $x$. **(2p)**