

**PHYS-E0461 Introduction to Plasma Physics for Fusion and Space Applications**  
**First examination 24.10.2018**

1. **(Gyro motion vs drifts)** (6p)

Calculate the Larmor radius, Larmor period and  $\mathbf{E} \times \mathbf{B}$  drift for an electron (energy 0.1 eV) in auroral ionosphere, where the magnetic field is 50 000 nT. Assume that the electric field is perpendicular to the magnetic field and has a magnitude of 20 mV/m. How far does the electron drift in one Larmor period? —

2. **(From kinetic to fluid approach)** (6p)

- (a) Calculate the first 2 velocity moments of the distribution function – particle density  $n_s$  and plasma flow  $n_s \mathbf{V}_s$  – when the velocity space dependence is given by the Maxwellian distribution (normalized to particle density).  
 (b) Derive the continuity equation

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{V}_s) = \frac{dn_s}{dt} + n_s \nabla \cdot \mathbf{V}_s = 0$$

by taking the first velocity moment of the Vlasov equation

$$\frac{\partial f_s}{\partial t} + \dot{\mathbf{r}} \cdot \nabla f_s + \dot{\mathbf{v}} \cdot \nabla_{\mathbf{v}} f_s = 0.$$

Hint: For the third term use the divergence theorem

$$\int_V dV \nabla \cdot \mathbf{A} = \int_S d\mathbf{S} \cdot \mathbf{A},$$

and remark that the integrand vanishes at the boundary  $S$ .

3. **(Magnetic bottle)** (6p)

Take a magnetic bottle with an axial field:  $B(z) = B_0[1 + (z/a)^2]$ . Sketch the field to align your  $z$  axis properly. Using conservation of energy and magnetic moment  $\mu$ , show that a particle (mass  $m$ ) that is mirroring between points  $-z_m$  and  $z_m$  has a longitudinal velocity given by

$$v_{\parallel} = \sqrt{\frac{2\mu B_0}{m}} \sqrt{\left(\frac{z_m}{a}\right)^2 - \left(\frac{z}{a}\right)^2}.$$

What is the particle velocity at the center of the bottle (where  $B = B_0$ ) and at the mirror points?

4. **(Whistling at low frequencies)** (6p)

Derive the dispersion relation

$$k = \frac{\omega_p}{c} \sqrt{\frac{\omega}{\Omega_e}}$$

for the whistler wave as the low-frequency part of the  $R$ -wave dispersion relation

$$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 - \Omega_e/\omega}.$$

Using the low-frequency dispersion relation, show that the group velocity of the whistler wave is

$$v_g r = \frac{2c\sqrt{\omega_e}}{\omega_p} \sqrt{\omega}$$

5. **(Essay: Waves in plasmas)** (6p)

Think about what you have learned about waves in plasmas. Explain why in plasmas there is such a host of different waves while in a neutral gas there is just the sound wave. Then try to organize the zoo of them in your head and describe as many as you remember. In particular, what role do things like background magnetic field, plasma density and temperature, and the frequency of the perturbation play in this game.