M

ELEC-E7210 Communication Theory

This is a closed book exam All five tasks are evaluated and taken into account in the grading. The exam can be written in Finnish, Swedish or English.

Exam 13.12. 2018

1. The mobile radio channel can be described by the coherence time, and the coherence bandwidth.
   a) What is the relation of the coherence bandwidth to the delay spread of the channel?
   b) Using the coherence bandwidth and the system bandwidth, how do you define a wideband (frequency selective) vs. narrow band (frequency flat) fading channel?
   c) Which technical solutions are needed when communicating over a frequency selective (but not frequency flat) channel?
   d) What is the relation of the coherence time to the Doppler spread of the channel, and what is the relation of Doppler spread to speed? $\frac{v\cos\theta \cdot f_c}{c}$
   e) Using the coherence time and the symbol period, how do you define a slowly vs. rapidly fading channel?
   f) Which solutions can be used when communicating over slow (but not rapid) fading channels?
   (Short answers to the six questions above are expected, at most a couple of sentences.)

2. Assume an OFDM transmission in a tapped delay line channel with two channel taps $[h_0 \ h_1] = [3 \ 1]$. Each transmission block of two symbols is prepended with a cyclic prefix (CP) of length one. In the block of interest, two QPSK symbols x_1 and x_2 are transmitted, in the previous block, symbols x_{-1} and x_0 were transmitted. The transmitted symbols in samples 1 and 2 are $\bar{\mathbf{x}} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \mathbf{M}^{\mathrm{H}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, with $\mathbf{M}^{\mathrm{H}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

   a) At the transmitter, the symbols are first processed by a inverse Discrete Fourier Transform. What are the transmitted signals at discrete time instances $[-1, 0, 1, 2]$ in terms of the QPSK symbols? The CP for the block of interest is transmitted in time instance 0.

   b) Write the 3-element vector of transmitted symbols for time instances $0, 1, 2$. Construct the channel matrix of Toeplitz form that gives the received samples at these instances, when multiplying the 3-element vector of transmitted signals. $y_m = \sum_{k} c_k h_{k-m}$

   c) After dropping the CP, consider the received samples in discrete time instances $[1, 2]$, the vector $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. Construct the circulant channel matrix $\mathbf{H}$ that describes the received signals in a signal model $\mathbf{y} = \mathbf{H}\bar{\mathbf{x}} + \mathbf{n}$, where the additive noise samples are $\mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$.

   d) At the receiver, the received signals are processed with a DFT matrix. What is the result after processing in terms of the transmitted symbols and the noise samples?

3. Consider Hybrid ARQ (HARQ) with Chase combining and at most one retransmission. If a coded transmission is erroneously received, the packet is retransmitted. We use a modulation/coding scheme(MCS) with transmission rate k, and an exponential packet error rate (PER) function

$$P_e(\gamma_p) = 10^{-\gamma_p/\gamma_c},$$

T

$1 + \sum_{t=0}^{T-1}$

where γ_p denotes the SNR of the packet, and γ_c is a constant which characterizes the MCS. We have a channel with a fixed per-transmission SNR γ. For the first transmission we have $\gamma_p = \gamma$. After a retransmission, we have $\gamma_p = 2\gamma$.

a) Calculate the residual packet error probability, i.e. the probability that a packet is in error after the two possible HARQ transmissions.

b) Calculate the expected number of transmissions when a single packet is attempted to be transmitted.

c) Calculate the expected throughput.

d) Take $\gamma = \gamma_c/3$. How much larger is the expected throughput for HARQ with at most $T = 2$ transmissions, compared to a single transmission without HARQ using the same MCS?

4. Consider a channel with M diversity branches (say M receive antennas) and i.i.d. Rayleigh fading on each branch, with white complex Gaussian noise. The instantaneous SNR γ of each of the M independent diversity branches comes from the PDF

$$f(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}}$$

where $\overline{\gamma}$ is the average SNR, same for all branches.

   a) Let us assume that $M = 1$ and that we are using BPSK and targeting bit error probability 10^{-3} or less. If the average SNR $\overline{\gamma}$ is 10dB what is the outage probability?

   b) Assume now that $M = 2$, $\overline{\gamma}$ is again 10dB. We are using selection combining, i.e. the system selects the branch with the highest SNR. What is the outage probability?

Note that for BPSK in AWGN channel the bit error probability can be approximated by $P_b \approx Q(\sqrt{2\gamma_b})$, where γ_b is the instantaneous SNR of the BPSK signal. We also have that $Q(3) = 0.00135$, $Q(3.1) = 0.00097$ and $Q(2.8) = 0.00256$.

5. Consider a static r-Tx and r-Rx antenna MIMO channel, with $r \times r$ channel matrix $\mathbf{H}$, known perfectly to Tx and Rx. Given codeword $\mathbf{x} = [x_1, x_2, \ldots, x_r]^T$, the Rx signal is modeled as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where the vector $\mathbf{n}$ is white Gaussian noise. Given matrix $\mathbf{H}$, the capacity of the channel is

$$C_{\mathbf{H}}(\rho) = B \max_{\rho_i:\, \sum_i \rho_i \le \rho} \sum_i \log_2(1 + \lambda_i \rho_i),$$

where ρ is average SNR, B is bandwidth, and $\sqrt{\lambda_i}$ are singular values of $\mathbf{H}$. Now take $B =$ 100 KHz, $\rho = 10$ dB, and $\mathbf{H} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

   a) Find the singular values of $\mathbf{H}$. Express the MIMO signal model with this channel matrix in an equivalent form in terms of parallel channels.

   b) Find the capacity of the channel with this channel matrix.

Hint: Note that

$$\log_2(1 + x) + \log_2(1 + y) \le 2 \log_2((1 + y + 1 + x)/2)$$

for all positive x and y.