MEC-E1020 Fluid dynamics - Exam - 24.10.2019

Write each task (1-4) on a separate piece of paper. Remember to write your name on your "cheat sheet" and to return them with the exam papers.

1 Fundamental equations and their solutions

Let's consider a steady, fully developed, two-dimensional, laminar and incompressible flow between two parallel plates, separated by a distance $h = 1\,\text{mm}$. The upper plate is moving with a speed $U = 3\,\text{m/s}$ and the lower plate is stationary. The pressure is constant everywhere. The fluid between the plates has a density $\rho = 817\,\text{kg/m}^3$ and a dynamic viscosity $\mu = 19.2 \times 10^{-3}\,\text{kg/(m s)}$. The influence of gravity can be neglected.

a) What can you say about the acceleration of a fluid particle in this case? Justify your answer. (1p)

b) By starting from the momentum equations show that in this case the net shear force acting on a differential fluid element has to be zero. When dropping out terms, justify this separately for each term. (2p)

c) Calculate the power per unit area required to move the upper plate. (1p)

d) Work is done to run the upper plate. What happens to this energy? Justify your answer with an appropriate analysis of the mechanical (kinetic) energy balance

$$\rho \frac{D}{Dt} \int_V \frac{1}{2} u_i^2 dV = \int_V \rho g_i u_i dV + \int_A u_i \tau_{ij} n_j dA + \int_V p \frac{\partial u_i}{\partial x_i} dV - \int_V \phi dV \ ,$$

where $\phi = 2\mu e_{ij} e_{ij}$. (2p)

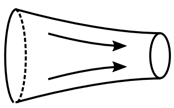
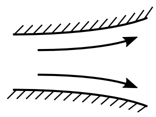
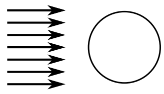
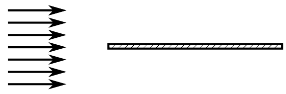
2 Boundary layers and related flows

a) When studying boundary layers we assume that the pressure does not vary in the wall normal direction. How can you justify this assumption? (1p)

b) What is the influence of streamline convergence or divergence on the boundary layer thickness? How can you justify this influence? (2p)

c) Consider a rectangular, thin, flat plate with an aspect ratio of 1:2. The plate is parallel to the flow, i.e. there is no pressure drag. Does the orientation of the plate (short or long edge along the direction of the flow) affect the drag and if yes, how? Note that the plate would be parallel to the flow in both cases. Justify your answer based on physical reasoning. (1p)

d) In which of the following flow cases there is a possible risk of boundary layer separation: converging nozzle, diffuser, circular cylinder with the axis normal to the flow, flat plate parallel to the flow? Justify your answer for all four cases (points are awarded only for cases with reasonable justifications). Sketches of the cases are shown below. (2p)

Converging nozzle	*Diffuser*	*Circular cylinder*	*Flat plate*
			

3 Instability and turbulence

a) Explain, what we mean by the energy cascade and what is the role of the viscosity in this? (2p)

b) Describe the structure of a wall-bounded turbulent boundary layer and the characteristic variation of the velocity as a function of the distance from the wall in the different parts of the layer. (2p)

c) Discuss the origin of the term $-\rho\overline{u_i u_j}$ in the Reynolds-averaged equations. Which physical process is the stress describing (justify your answer)? (2p)

4 Numerical techniques

a) Explain, what we mean by numerical diffusion and numerical dispersion. (2p)

b) Which are the pros and cons of the explicit and implicit schemes? (2p)

c) Show that the centered scheme for the convective term

$$U\frac{\partial T}{\partial x}$$

leads to a second order accurate discretisation. (2p)