MEC-E1020 Fluid dynamics - Exam - 24.10.2019

The general guidelines for the assessment are specified for each task below. These guidelines will be adapted on a case by case basis. In order to pass, you need to get at least 8 points in total.

1 Fundamental equations and their solutions

Let's consider a steady, fully developed, two-dimensional, laminar and incompressible flow between two parallel plates, separated by a distance $h = 1\,\mathrm{mm}$. The upper plate is moving with a speed $U = 3\,\mathrm{m/s}$ and the lower plate is stationary. The pressure is constant everywhere. The fluid between the plates has a density $\rho = 817\,\mathrm{kg/m^3}$ and a dynamic viscosity $\mu = 19.2 \times 10^{-3}\,\mathrm{kg/(m\,s)}$. The influence of gravity can be neglected.

a) What can you say about the acceleration of a fluid particle in this case? Justify your answer. (1p)

b) By starting from the momentum equations show that in this case the net shear force acting on a differential fluid element has to be zero. When dropping out terms, justify this separately for each term. (2p)

c) Calculate the power per unit area required to move the upper plate. (1p)

d) Work is done to run the upper plate. What happens to this energy? Justify your answer with an appropriate analysis of the mechanical (kinetic) energy balance

$$\rho \frac{D}{Dt} \int_V \frac{1}{2} u_i^2 dV = \int_V \rho g_i u_i dV + \int_A u_i \tau_{ij} n_j dA + \int_V p \frac{\partial u_i}{\partial x_i} dV - \int_V \phi dV\ ,$$

where $\phi = 2\mu e_{ij} e_{ij}$. (2p)

Assessment

- Have a look at section 9.4 and the solution for the assignments on week 1 and 2.

Subtask a

- Acceleration of a fluid particle correct (zero). +0.5
- The result for the acceleration justified based on the assumptions of steady (+0.25) and fully developed (+0.25) flow.

Subtask b

- You can assume that there is only one relevant velocity component or you can derive this using the continuity condition and the impermeablity condition on the plate surfaces.
- Equations correctly simplified (only one viscous term left in the final form). +0.5

- Simplification of each term justified correctly. +1
- The remaining viscous term correctly associated with the net viscous force. +0.5

Subtask c

- Shear stress can be calculated by first solving the velocity profile from the simplified momentum equation.
- Shear stress on the upper plate calculated correctly (U/h). +0.5
- Power calculated correctly as a product of plate velocity and shear stress ($173\ \mathrm{W/m^2}$). +0.5

Subtask d

- The link between the work done and the generation of heat explained. +0.5
- Mechanical energy simplified correctly (stress work and viscous dissipation left). +0.5
- Stress work correctly associated with the work done to run the plate. +0.5
- Viscous dissipation associated to the heat generated and shown to be equal to the rate of work done to run the plate. +0.5

2 Boundary layers and related flows

a) When studying boundary layers we assume that the pressure does not vary in the wall normal direction. How can you justify this assumption? (1p)

   - Have a look at section 10.2 in Kundu and Cohen and slide 14 in the slide set 3.2.
   - The answer refers to the simplification of the y-momentum equation in case of boundary layer flows. +1

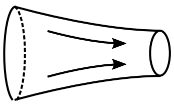
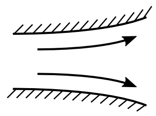
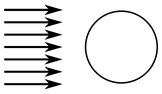
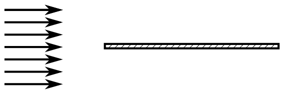
b) What is the influence of streamline convergence or divergence on the boundary layer thickness? How can you justify this influence? (2p)

   - Have a look at slides 15 and 17 in slide set 3.1.
   - The influence of convergence and divergence is correctly described. +1
   - The justification is linked to the deformation of the cross-section of the fluid element. +1

c) Consider a rectangular, thin, flat plate with an aspect ratio of 1:2. The plate is parallel to the flow, i.e. there is no pressure drag. Does the orientation of the plate (short or long edge along the direction of the flow) affect the drag and if yes, how? Note that the plate would be parallel to the flow in both cases. Justify your answer based on physical reasoning. (1p)

   - Have a look at pages 357-358 and Figure 10.9 in Kundu and Cohen.
   - The larger drag is correctly identified to occur in the case, in which the short edge is along the flow direction. +0.5
   - The reason for the change in the drag is related to the evolution of the shear stress along the length of the plate. +0.5

d) In which of the following flow cases there is a possible risk of boundary layer separation: converging nozzle, diffuser, circular cylinder with the axis normal to the flow, flat plate parallel to the flow? Justify your answer for all four cases (points are awarded only for cases with reasonable justifications). Sketches of the cases are shown below. (2p)

Converging nozzle	*Diffuser*	*Circular cylinder*	*Flat plate*
			

   - Have a look at sections 10.7-8 in Kundu and Cohen.
   - The cases with possible separation are identified to be the diffuser and circular cylinder cases and the possibility is justified for each of the four cases based on the corresponding pressure gradient. +4*0.5

3 Instability and turbulence

a) Explain, what we mean by the energy cascade and what is the role of the viscosity in this? (2p)

- Have a look at section 13.8 in Kundu and Cohen
- The transfer of energy from the mean flow to eddies of various sizes explained. +1
- The role of viscosity in the dissipation of the turbulent energy explained. +1

b) Describe the structure of a wall-bounded turbulent boundary layer and the characteristic variation of the velocity as a function of the distance from the wall in the different parts of the layer. (2p)

- Have a look at section 13.11 in Kundu and Cohen and the assignment for week 4.
- The four parts of the turbulent boundary layer identified (+1) and the variation of the velocity in each part briefly explained (+1).

c) Discuss the origin of the term $-\rho\overline{u_i u_j}$ in the Reynolds-averaged equations. Which physical process is the stress describing (justify your answer)? (2p)

- Have a look at section 13.5 (particularly pages 550-552) in Kundu and Cohen.
- The origin of the stress is correctly related to the nonlinear convection terms (+0.5) in the averaged momentum equations (+0.5).
- The Reynolds stress is correctly related to the transfer of mean momentum due to fluctuations (+0.5) and this is sensibly justified (+0.5).

4 Numerical techniques

a) Explain, what we mean by numerical diffusion and numerical dispersion. (2p)

- Have a look at section 7.1 in Hirsch and slide 22 in the slide set 5.2.
- Numerical diffusion is sensibly described. +1
- Numerical dispersion is sensibly described. +1

b) Which are the pros and cons of the explicit and implicit schemes? (2p)

- Have a look at section 11.2 in Kundu and Cohen and slides 14 and 16 in slide set 5.2.
- The pros and cons related to stability (+1) and complexity of implementation (+1) have been identified correctly.

c) Show that the centered scheme for the convective term

$$U\frac{\partial T}{\partial x}$$

leads to a second order accurate discretisation. (2p)

- Have a look at section 11.2 in Kundu and Cohen.
- The centered scheme is correct. +0.5
- Taylor series expansion is applied for the discrete terms. +0.5
- The expanded form is correctly derived. +0.5
- The result is correctly used to show that the scheme is second order accurate. +0.5