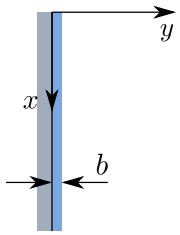
MEC-E1020 Fluid dynamics - Exam - 25.10.2018

Write each task (1-4) on a separate piece of paper. Remember to write your name on your "cheat sheet" and to return them with the exam papers.

1 Fundamental equations and their solutions

Let's consider a thin film of water flowing down a vertical wall. The flow is incompressible, two-dimensional and fully developed. The thickness b of the film is constant. The shear stress between the film and the surrounding air is negligible. The pressure acting on the interface between the film and the air is atmospheric everywhere.



a) By starting from the continuity condition and the momentum equations for incompressible flow determine the distribution of velocity within the film. When you drop out terms from the equations, justify this accordingly. (3p)

b) Which forces are acting on a differential fluid element within the film? How does the shear stress vary within the film and how is this related to the balance of the forces? (1p)

c) Gravity is doing work on the fluid within the film. What happens to this energy? Justify your answer with an appropriate analysis of the mechanical (kinetic) energy balance

$$\rho \frac{D}{Dt} \int_V \frac{1}{2} u_i^2 dV = \int_V \rho g_i u_i dV + \int_A u_i \tau_{ij} n_j dA + \int_V p \frac{\partial u_i}{\partial x_i} dV - \int_V \phi dV \ ,$$

where $\phi = 2\mu e_{ij} e_{ij}$. (2p)

2 Boundary layers and related flows

a) It is known that the natural rate of growth for a laminar boundary layer is $\delta \sim x/\sqrt{\mathrm{Re}_x}$. Where is this result coming from? (1p)

b) Discuss the influence of longitudinal and transversal pressure gradients on the boundary layer flow. (2p)

c) Discuss the influence of a boundary layer on the flow outside of the boundary layer and on the pressure field on the body surface. (1p)

d) Explain, how we can derive the boundary layer equations and discuss the main differences between the boundary layer equations and the full Navier-Stokes equations. (2p)

3 Instability and turbulence

a) What do we mean by y^+ and u^+? (1p)

b) Discuss the process of production, cascade and dissipation of turbulent kinetic energy. (3p)

c) Discuss the origin of the Reynolds stress. Which physical process is the stress describing (justify your answer)? (2p)

4 Numerical techniques

A discretised scheme is given as

$$T_i^{n+1} + \left(\frac{3U\Delta t}{2\Delta x} - 1\right) T_i^n - \frac{2U\Delta t}{\Delta x} T_{i-1}^n + \frac{U\Delta t}{2\Delta x} T_{i-2}^n = 0 \, .$$

where U is assumed to be a positive velocity. The subscripts refer to different points of the spatial discretisation and the superscripts to the discrete time levels.

a) Is this an explicit or implicit scheme? Justify your answer. (1p)

b) Determine the original, continuous equation and the truncation error? What is the order of the scheme? (3p)

c) Show that the scheme is unstable for any choice of the time step Δt and therefore useless. (2p)