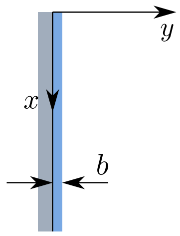
MEC-E1020 Fluid dynamics - Exam - 25.10.2018

The general guidelines for assessment are specified for each task below. These guidelines will be adapted on a case by case basis. In order to pass, you need to get at least 8 points in total.

1 Fundamental equations and their solutions

Let's consider a thin film of water flowing down a vertical wall. The flow is incompressible, two-dimensional and fully developed. The thickness b of the film is constant. The shear stress between the film and the surrounding air is negligible. The pressure acting on the interface between the film and the air is atmospheric everywhere.



a) By starting from the continuity condition and the momentum equations for incompressible flow determine the distribution of velocity within the film. When you drop out terms from the equations, justify this accordingly. (3p)

b) Which forces are acting on a differential fluid element within the film? How does the shear stress vary within the film and how is this related to the balance of the forces? (1p)

c) Gravity is doing work on the fluid within the film. What happens to this energy? Justify your answer with an appropriate analysis of the mechanical (kinetic) energy balance

$$\rho \frac{D}{Dt} \int_V \frac{1}{2} u_i^2 dV = \int_V \rho g_i u_i dV + \int_A u_i \tau_{ij} n_j dA + \int_V p \frac{\partial u_i}{\partial x_i} dV - \int_V \phi dV \ ,$$

where $\phi = 2\mu e_{ij} e_{ij}$. (2p)

Assessment

- Have a look at section 9.4 and the solution for the assignments on week 1 and 2, particularly for the consideration of the energy balance.

Subtask a

- You can assume that there is only one relevant velocity component or you can derive this using the continuity condition and the impermeablity condition on the wall surface.

- Equations correctly simplified. +0.5
- Simplification of each term justified correctly. +1
- Velocity profile solved from the resulting equation by integrating twice. +0.5
- Integration constants determined using the no-slip and zero shear stress conditions. +2*0.25
- Velocity profile correct. +0.5

Subtask b

- Correct force contributions identified. +2*0.25
- The variation of the shear stress correct. +0.25
- The variation of the shear stress linked to the force balance. +0.25

Subtask c

- The link between the work done and the generation of heat explained. +0.5
- Mechanical energy equation simplified correctly. +0.5
- Total stress work treated correctly. +0.5
- Correct balance of work demonstrated. +0.5

2 Boundary layers and related flows

a) It is known that the natural rate of growth for a laminar boundary layer is $\delta \sim x/\sqrt{\mathrm{Re}_x}$. Where is this result coming from? (1p)

- Have a look at section 10.2 (particularly page 342) in Kundu and Cohen.
- The answer is sensibly discussing the balance of the inertia and viscous terms. +1

b) Discuss the influence of longitudinal and transversal pressure gradients on the boundary layer flow. (2p)

- Have a look at sections 10.7, 10.8 and 10.13 in Kundu and Cohen.
- The influence of the longitudinal gradient on the evolution of the boundary layer thickness (+0.5) and possible separation (+0.5) discussed.
- The influence of the transversal gradient on cross-flow discussed (+0.5) paying attention to the link between the varying flow speed and streamline curvature (+0.5).

c) Discuss the influence of a boundary layer on the flow outside of the boundary layer and on the pressure field on the body surface. (1p)

- Have a look at section 10.3 in Kundu and Cohen and the description slide set of week 3.
- The displacement effect is discussed. +0.5
- The influence of the displacement effect on the pressure field is discussed. +0.5

d) Explain, how we can derive the boundary layer equations and discuss the main differences between the boundary layer equations and the full Navier-Stokes equations. (2p)

- Have a look at section 10.2 in Kundu and Cohen.
- The process of simplifying the Navier-Stokes equations based on the relative order of magnitude of the different terms discussed. +0.5
- The absence of one viscous term in the x-momentum equation (+0.5) and the role of the y-momentum equation as a condition for pressure discussed (+0.5).
- The change in the character of the equations sensibly discussed. +0.5

3 Instability and turbulence

a) What do we mean by y^+ and u^+? (1p)

- Have a look at section 13.11 in Kundu and Cohen and the slide set of week 4.2.
- Correct definition of y^+. +0.5
- Correct definition of u^+. +0.5

b) Discuss the process of production, cascade and dissipation of turbulent kinetic energy. (3p)

- Have a look at section 13.8 in Kundu and Cohen.
- The production of turbulent kinetic energy is correctly related to the interaction between the mean field and the largest eddies (+0.5) through the vortex stretching process (+0.5).
- The cascade process is correctly related to the interaction of the larger and smaller eddies (+0.5) through the vortex stretching process (+0.5).
- The dissipation of the turbulent kinetic energy is correctly related to the process of turning turbulent kinetic energy into heat (+0.5) at the smallest scales due to viscous effects (+0.5).

c) Discuss the origin of the Reynolds stress. Which physical process is the stress describing (justify your answer)? (2p)

- Have a look at section 13.5 (particularly pages 550-552) in Kundu and Cohen.
- The origin of the stress is correctly related to the nonlinear convection terms (+0.5) in the averaged momentum equations (+0.5).
- The Reynolds stress is correctly related to the transfer of mean momentum due to fluctuations (+0.5) and this is sensibly justified (+0.5).

4 Numerical techniques

A discretised scheme is given as

$$T_i^{n+1} + \left(\frac{3U\Delta t}{2\Delta x} - 1\right) T_i^n - \frac{2U\Delta t}{\Delta x} T_{i-1}^n + \frac{U\Delta t}{2\Delta x} T_{i-2}^n = 0 \; .$$

where U is assumed to be a positive velocity. The subscripts refer to different points of the spatial discretisation and the superscripts to the discrete time levels.

a) Is this an explicit or implicit scheme? Justify your answer. (1p)

b) Determine the original, continuous equation and the truncation error? What is the order of the scheme? (3p)

c) Show that the scheme is unstable for any choice of the time step Δt and therefore useless. (2p)

Assessment

- Have a look at section 11.2 in Kundu and Cohen and the exercise on week 5.

Subtask a

- The type of the scheme is correct. +0.5
- The type of the scheme is justified based on the number of unknowns in the discretised equation. +0.5

Subtask b

- Taylor series expansion is applied correctly for the future as well as the first and the second neighbour values. +3*0.5
- Taylor series expansion is substituted into the discrete equation and equation cleaned up to reveal the original equation and the truncation error. +0.5
- The original equation and the truncation error are correct. +2*0.25
- The order of the scheme in time and space is correct. +2*0.25

Subtask c

- The temporal derivatives are transformed into spatial derivatives. +0.5
- A modified equation is constructed based on these. +0.5
- The modified equation is correct. +0.5
- The instability is shown by studying the sign of the numerical diffusion coefficient. +0.5