MEC-E1020 Fluid dynamics - Exam - 14.12.2018

Remember to write your name on your ”cheat sheet” and to return them with the exam papers.

1 Fundamental equations and their solutions

Consider a fully developed laminar flow of a Newtonian fluid between two parallel plates. The plate at $y = 0$ is stationary, whereas the plate at $y = b$ is moving with speed U in the direction of the positive x-axis. The velocity field is given by

$$u(y) = U\frac{y}{b}, \quad v = 0, \quad w = 0 \ .$$

Assume that the influence of gravity can be neglected and that the pressure is uniform everywhere.

a) Calculate the force per unit area required to pull the upper plate. (1p)

b) Show that the volume of a differential fluid element does not change in this case. (1p)

c) Discuss the balance of forces acting on a differential fluid element in this case. (1p)

d) Discuss the balance of energy in this case and support your claims by appropriate analysis of the balance of the mechanical energy

$$\rho\frac{D}{Dt}\int_V \frac{1}{2}u_i^2 dV = \int_V \rho g_i u_i dV + \int_A u_i \tau_{ij} n_j dA + \int_V p\frac{\partial u_i}{\partial x_i} dV - \int_V 2\mu e_{ij} e_{ij} dV \ . \text{(3p)}$$

2 Boundary layers and related flows

a) By how much does the laminar boundary layer thickness at the end of a flat plate and the drag of the plate change and why, if

- the velocity doubles, (1p)
- the length of the plate doubles. (1p)

Assume in both cases that the flow remains laminar.

b) Let’s consider the flow over an axisymmetric body. How does the thickness of the boundary layer change as the radius of the body

- increases in the downstream direction or
- decreases in the downstream direction?

Justify your answer. (2p)

c) Explain, how a pressure gradient in the direction of the flow affects the shape of the velocity profile inside a boundary layer. How can this effect be explained based on the momentum equation on the wall? (2p)

3 Instability and turbulence

a) A point is at $y^+ = 100$ from the wall. In which part of a turbulent boundary layer do you expect this point to be? Justify your answer. (1p)

b) Explain, what is the link between the equations for the mean kinetic energy and for the turbulent kinetic energy, i.e. what is the common term between these two equations and what physical process does this term represent. (2p)

c) Discuss the problems related to the direct solution of the Navier-Stokes equations in case of turbulent flows and how turbulent flows are typically solved. (3p)

4 Numerical techniques

a) Show that the backward scheme for the convective term

$$U \frac{\partial T}{\partial x}$$

leads to a first order accurate discretisation. (2p)

b) Explain, what we mean by numerical diffusion and numerical dispersion. (2p)

c) An explicit scheme for the pure convection equation is given as

$$\frac{f_i^{n+1} - f_i^n}{\Delta t} + U \frac{f_i^n - f_{i-1}^n}{\Delta x} = 0 \ .$$

What would be the corresponding implicit scheme with the same spatial discretisation? What is the benefit of using an implicit scheme instead of an explicit scheme? (2p)